CMPS 6640/4040 Computational Geometry – Spring 16

3/10/16

4. Homework

Due **3/31/16** before class

1. **Levels (13 points)**
   (a) (2 points) Draw an example of a simple arrangement of six lines. Annotate each vertex, edge, and face of the arrangement with its level.
   (b) (3 points) Is it possible for a simple arrangement of n lines to have no vertex of level $n-2$? Justify your answer.
   (c) (4 points) Describe how one can compute the level of each face in an arrangement during the incremental construction of the arrangement.
   (d) (4 points) Describe how one can compute the level of each face in an arrangement during the sweep-line construction of the arrangement.

2. **Minimum-Area Triangle (7 points)**
   Let P be a set of n points in the plane. Give an algorithm to compute the minimum area triangle whose vertices are chosen from points in P. Analyze the runtime. *(Hint: Use the dual line arrangement. The area of a triangle is base times height divided by two.)*

3. **Sweeping Triangles (10 points)**
   Let S be a set of n disjoint triangles in the plane. We want to find a set of $n-1$ line segment "bridges" that have the following properties:

   - Each bridge connects a point on the boundary of one triangle to a point on the boundary of another triangle.
   - No bridge intersects another bridge or any of the triangles.
   - All $n-1$ bridges together connect all triangles to each other. That is, the triangles and bridges together form a connected graph.

   Develop an $O(n \log n)$ time plane sweep algorithm to solve this problem. Describe the sweep line status, the events, and what actions are performed at each event.

4. **Arrangements (10 points)**
   Let L be a set of n lines in the plane, and let $\mathcal{A}(L)$ be its arrangement. For a line l let $z(L,l)$ be the complexity of the zone of l in $\mathcal{A}(L)$. For a face $f \in \mathcal{A}(L)$ let n_f be the number of edges on f (which equals the number of vertices on f).
   (a) (8 points) Prove $\sum_{f \in \mathcal{A}(L)} 2\binom{n_f}{2} \in O(\sum_{l \in L} z(L \setminus \{l\}, l))$ using a combinatorial argument.
   (b) (2 points) Use the result from (a) to prove that $\sum_{f \in \mathcal{A}(L)} n_f^2 \in O(n^2)$

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5. **Convex Hull of Intersections (10 points)**
Let $\mathcal{L}$ be a set of n lines in the plane, no two of which are parallel. Let S be the set of all $O(n^2)$ intersection points between any two lines in $\mathcal{L}$.

   (a) (6 points) Give an $O(n \log n)$ time algorithm to compute an axis-parallel rectangle that contains S.

   (b) (4 points) Give an $O(n \log n)$ time algorithm that computes $CH(S)$.

   (Hint: Your algorithms cannot compute all points in S explicitly. Sort all lines by slope, and prove that it is enough to consider only a certain subset of intersection points.)