CMPS 6640/4040 Computational Geometry – Spring 16

2/26/16

3. Homework

Due **3/10/16** before class

1. **Maintenance of Delaunay Triangulations (10 points)**
   Let P be a set of n points in the plane, for which a Delaunay Triangulation has already been constructed using randomized incremental construction. You are given all data structures that were used to compute the Delaunay Triangulation.

   (a) (5 points) Given a point $q \in P$, give an algorithm that *deletes* q from the Delaunay Triangulation, i.e., it computes the $DT(P \setminus \{q\})$. Analyze the runtime. The runtime should depend on $k = \deg(q, DT(P))$.

   (b) (5 points) Given another point $q \notin P$, give an algorithm that *adds* q to the Delaunay Triangulation, i.e., it computes $DT(P \cup \{q\})$. What is the runtime, expressed in n and k, where $k = \deg(q, DT(P \cup \{q\}))$?

2. **Backwards Analysis (10 points)**
   Consider the following algorithm:

```
FindMax(A,n){
  // Finds maximum in set A of n numbers
  if(n==1) return the single number in A
  else {
    x = extract random element from A // in constant time; x is removed from A
    y = FindMax(A,n-1)
    if(x<=y) return y;
    else
       Compare x with all remaining elements in A and return the maximum
  }
}
```

   (a) (4 points) Argue that this algorithm is correct, and give its worst-case runtime. (The runtime is proportional to the number of comparisons made.)

   (b) (6 points) Compute the expected runtime of this algorithm.
   (Hint: Introduce an indicator random variable for executing the else branch in the i-th step, and use backwards analysis to simplify the analysis.)

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3. **Facility Location (10 points)**
Suppose you are given a set P of n grocery stores in the plane.

   (a) (5 points) The task is to compute the location q of a new grocery store that maximizes the distance to all other grocery stores, i.e., such that the radius of an empty disk centered at q is maximized. Because the grocery store is supposed to be located within the city, we have the additional constraint that q has to lie inside $CH(P)$. Show how to compute q and analyze the runtime. *(Hint: Use Voronoi diagrams. Pay special attention to the case when q might lie on the boundary of $CH(P)$.)*

   (b) (5 points) Now we want to find a good location q for a wholesale distribution center that is reachable by all grocery stores. So, the task is to compute the center q of the smallest enclosing disk of P, i.e., the disk with smallest radius such that all points in P are contained in the disk. Show how to compute q and analyze the runtime.
   (Hint: Use a farthest-point Voronoi diagram.)

4. **Voronoi and Lifting (10 points)**
We saw in class that the Voronoi diagram of a set of points in $\mathbb{R}^2$ is the projection of the upper envelope of the dual lifted set of planes in $\mathbb{R}^3$. What does the projection of the *lower* envelope correspond to? Similarly, what does the projection of the *upper* convex hull of the points lifted to $\mathbb{R}^3$ correspond to?

   Answer these questions by researching on the internet. Cite the source you were using and give an explanation in your own words.

5. **Reverse Voronoi (10 points)**
Suppose we are given a subdivision of the plane into n convex regions. We suspect that this subdivision is a Voronoi diagram, but we do not know the sites. Develop an algorithm that finds a set of n point sites whose Voronoi diagram is exactly the given subdivision, if such a set exists.