CMPS 6640/4040 Computational Geometry – Spring 16

2/4/16

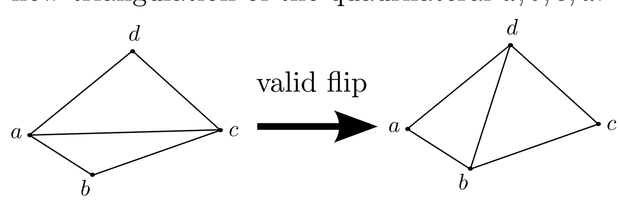
2. Homework

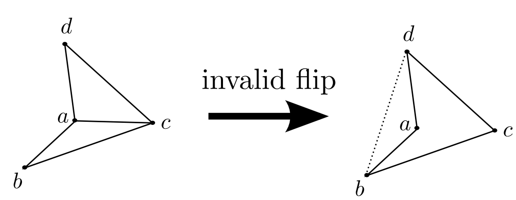
Due **2/18/16** before class

1. **Simple Point Location (10 points)**
Assume you are given a planar subdivision in a DCEL. (You may assume that the planar subdivision does not contain any holes, i.e., there are no nested faces.) Describe an algorithm that for a given point p in the plane finds the face in the subdivision that contains it. Your algorithm should run in $O(n)$ time. You do not have to write pseudo-code, but please make clear what DCEL operations you are using. Also please make sure the analysis is detailed enough to justify the $O(n)$ runtime clearly.

2. **RIC (10 points)**
   (a) (7 points) Describe and analyze an algorithm that computes the convex hull of a set of n points in the plane using randomized incremental construction in expected $O(n \log n)$ time. For this problem you are welcome to find an algorithm and its analysis on the web, but please describe it concisely in your own words and make the analysis very concise. Where does the log-factor come from?
   (b) (3 points) Give an example of a set of points in the plane, and a particular input order, that causes the convex hull algorithm to run in $O(n^2)$ when the points are added in this particular order. Make sure it is clear how your example generalizes to arbitrary values of n.

3. **Edge Flips (10 points)**
Consider a triangulated quadrilateral a, b, c, d in the plane, with diagonal $\overline{ac}$. An *edge flip* replaces $\overline{ac}$ with $\overline{bd}$. We only consider *valid edge flips* that yield a valid new triangulation of the quadrilateral a, b, c, d.





Show that any two triangulations of a convex polygon can be transformed into each other by edge flips.

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4. **Dobkin-Kirkpatrick (10 points)**
   Let P be a polytope on n vertices in $\mathbb{R}^3$, and assume Dobkin-Kirkpatrick's hierarchy has been computed for P. Describe and analyze an algorithm that for a given query point $q \in R^3$ computes the smallest distance to any point on P in $O(\log n)$ time. (You can assume the existence of basic geometric primitives, such as distance computation from a point to a plane or to a triangle).

5. **Linear-Time Convex Hull (10 points)**
   Let $(x_1, y_1), \ldots, (x_n, y_n)$ be the vertices of a convex polygon in $\mathbb{R}^2$, listed in counterclockwise order. And let $z_1, \ldots, z_n \mathbb{R}$ be n arbitrary numbers. Describe and analyze an algorithm that computes the convex hull of the points $(x_1, y_1, z_1), \ldots, (x_n, y_n, z_n)$ in expected $O(n)$ time.