CMPS 6640/4040 Computational Geometry – Spring 16

1/21/16

1. Homework

Due **2/4/16** before class

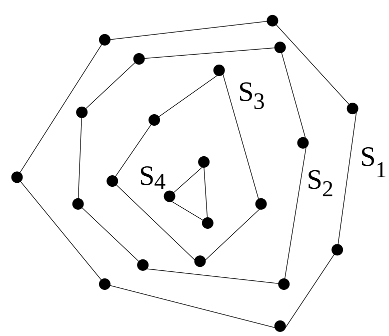
1. **Binary search (10 points)**
   Assume you have an orientation test available which can determine in constant time whether three points make a left turn (i.e., the third point lies on the left of the oriented line described by the first two points) or a right turn. Now, let a point q and a convex polygon $P = \{p_1, \ldots, p_n\}$ in the plane be given, where the points of P are stored in an array in counter-clockwise order around P. Give pseudo-code to determine an upper tangent from q to P in $O(\log n)$ time.

2. **Convex Hulls (10 points)**
   Let S be a set of n line segments in the plane. Let P be the set of $2n$ endpoints of the segments in S. Prove that the convex hull of S is exactly the same as the convex hull of P.

3. **Nested Convex Hulls (10 points)**
   Given a set S of n points in the plane, consider the subsets

$$
\begin{aligned}
S_1 &= S, \\
S_2 &= S_1 \setminus \{\text{set of vertices of } conv(S_1)\} \\
&\;\;... \\
S_i &= S_{i-1} \setminus \{\text{set of vertices of } conv(S_{i-1})\}
\end{aligned}
$$

   until S_k has at most three elements. Give an $O(n^2)$ time algorithm that computes all convex hulls $conv(S_1), conv(S_2), \ldots$. (If you can do it faster you can earn extra credit.)



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4. **Chan's Algorithm (10 points)**
Consider Chan's convex hull algorithm to compute the convex hull of a set P of n points in the plane. The main algorithm is as follows:

(1) $h^* = 2$; $L =$ fail
(2) while ($L \neq$ fail)
  (a) $h^* = \min(2^{2^i}, n)$
  (b) $L =$RestrictedHull(P, h^*)
  (c) i++
(3) return L

Let h be the number of vertices on the convex hull of P. If $h \leq h^*$ then RestrictedHull(P, h^*) returns the convex hull of P, otherwise it returns "fail".

For each of the two cases below, determine the big-Oh runtime of Chan's algorithm when replacing line (2)(a) with the shown expression. Justify your answers.

(a) $h^* = \min(i^2, n)$
(b) $h^* = \min(2^{2^{2^i}}, n)$

5. **Convex Sets (10 points)**
Consider *Radon's theorem:* Any set S of at least $d + 2$ points in $\mathbb{R}^d$ can be split into two subsets S_1 and S_2 such that $conv(S_1) \cap conv(S_2) \neq \emptyset$.

Assuming Radon's theorem is given, prove the following theorem:
Let $\{C_1, \ldots, C_n\}$ be a family of n convex sets in $\mathbb{R}^d$. Show that if any $d+1$ convex sets have a non-empty intersection, then so does the whole family.
(Hint: One possible proof goes by induction on n. By induction we know that there is a point p_i in the intersection $\bigcap_{j \neq i} C_j$, for all $i = 1, \ldots, n$. Then use Radon's theorem on the set $\{p_i \mid i = 1, \ldots, n\}$ to construct a point p that belongs to all the sets C_i.)