11/12/18

9. Homework

Due **11/19/18** at the beginning of class

Justify all your answers.

1. **Flow Updates (6 points)**
   Suppose you are given a flow network $G = (V, E)$ with source s, sink t, and non-negative integer capacities c. You are also given a maximum flow f on G.

   We would like to be able to make some changes to the flow network and update the maximum flow efficiently. In particular, we would like to support the following operations:

   - INCREMENT(e) increments the capacity of the edge $e \in E$ by 1.
   - DECREMENT(e) decrements the capacity of the edge $e \in E$ by 1.

   All operations maintain a valid flow network at all times.

   Give **efficient** algorithms for INCREMENT(e) and for DECREMENT(e) and analyze their complexities. Your algorithms should be faster than simply recomputing the maximum flow.

2. **Bipartite Matching (6 points)**
   Let $G = (V = L \cup R, E)$ be a bipartite graph, and let $G' = (V' = L \cup R \cup \{s, t\}, E')$ be the flow network corresponding to it.

   What is the maximum possible length (in terms of number of edges) that an augmenting path in the residual network of G' can have? Express your answer in terms of $|L|$ and $|R|$.

3. **Weighted Min-Cut (9 points)**
   Let $G = (V, E)$ be an undirected graph with positive edge weights $w : E \to \mathbb{R}^+$, and the goal is to find a cut with total minimum edge weight. In this exercise we will see that Karger's algorithm can indeed easily be adapted to work in this weighted setting as well.

   (a) Give an algorithm to select a random edge e of E, where the probability of choosing e is proportional to $w(e)$. *(Hint: Use a cumulative distribution function.)*

   (b) Let GUESSMINCUTWEIGHTED be the modification of GUESSMINCUT that uses part (a) to choose edges randomly proportional to their weights.

      i. What is the runtime of GUESSMINCUTWEIGHTED?

      ii. Prove that the probability of GUESSMINCUT computing a correct minimum cut in this weighted case is still $\Omega(1/n^2)$.