CMPS 6610 Algorithms – Fall 18

10/29/18

8. Homework

Due **11/5/18** at the beginning of class

Justify all your answers.

1. **Fibonacci Heaps (6 points)**
   (a) Describe what sequence of operations in a Fibonacci heap results in a root that is marked.
   (b) For any $n \geq 1$, describe how to form a sequence of Fibonacci heap operations that creates a Fibonacci heap in which all n nodes form a single path of height n.
   (c) In the amortized analysis of Fibonacci heaps, why is the potential function not simply $\Phi(H) = trees(H) + marks(H)$?

2. **Negative edge weights (4 points)**
   (a) Give an example of a directed connected graph with real edge weights (that may be negative) for which Dijkstra's algorithm produces incorrect answers.
   (b) Suppose the weighted, directed graph $G = (V, E)$ has a special structure in which edges that leave the source vertex s may have negative weights. All other edge weights are nonnegative, and there are no negative-weight cycles. Show that Dijkstra's algorithm correctly finds shortest paths from s in G.

3. **Floyd-Warshall (6 points)**
   (a) Show how to use the Floyd Warshall algorithm to detect whether a weighted graph contains a negative weight cycle.
   (b) Suppose you run the Floyd Warshall algorithm for $k = 1$ to $n-1$, and not to n. Does this still compute the correct output?
   (c) In the Floyd Warshall algorithm, can you switch the order of the three for-loops and still compute the correct output?

4. **Shortest Path with Minimum Number of Edges (8 points)**
   Let $G = (V, E)$ be a directed graph with positive edge weights, and let $s \in V$ be a source vertex.
   (a) Give an example which shows that Dijkstra's algorithm does not always compute shortest paths from s with the minimum number of edges. Indicate the d-values and the shortest path tree.
   (b) Modify Dijkstra's algorithm to compute shortest paths from s with the minimum number of edges.
   (c) Show the output (d-values and shortest path tree) of your modified algorithm on the example from part (a) above.