CMPS 6610 Algorithms – Fall 18

9/24/18

4. Homework

Due **10/1/18** at the beginning of class

1. **Lower bound (4 points)**
   Show that the expected runtime for randomized Quicksort of n distinct numbers is in $\Omega(n \log n)$.

2. **Quicksort with duplicates (10 points)**
   (a) Consider running deterministic quicksort on an array with n equal values. How does deterministic quicksort behave in this case, and what is its runtime? What are the behavior and the runtime of randomized quicksort on such an array?
   (b) How does deterministic quicksort behave on an array of length n that contains only two different values?
   (c) Give pseudo-code for a 3-WAY-PARTITION routine that partitions the array into three parts: values less than the pivot, values equal to the pivot, values greater than the pivot. Your code should be in-place (so it should use at most constant extra storage) and it should run in linear time.
   (d) Consider an implementation of quicksort which uses 3-WAY-PARTITION and only recurses on those portions of the array that contain values less than or greater than the pivot. If the array of length n contains only 2 different values, what is the worst-case runtime?
   (e) If the array of length n contains d different values, show that the worst-case runtime of this variant of quicksort that uses 3-way partition is $O(dn)$.

FLIP OVER TO BACK PAGE $\Longrightarrow$

3. **Randomized code snippets (8 points)**
Consider the following code snippets, where `RandomInteger(i)` takes $O(1)$ time and returns an integer between 1 and i uniformly at random (i.e., each with probability $1/i$).

(a)
```
for(i=2; i<=n; i++){
  r=RandomInteger(i);
  if(r==1){
    for(j=1; j<=n; j++){
      for(k=1; k<=n; k++){
        print(''Roll'');
      }
    }
  }
```

(b)
```
for(i=2; i<=n; i++){
  r=RandomInteger(n);
  if(r==1){
    for(j=1; j<=n; j++){
      for(k=1; k<=n; k++){
        print(''Wave'');
      }
    }
  }
```

Answer the following questions for each of the code snippets above.

(a) (2 points) What is the best case runtime, in terms of n, of this code snippet? Describe what triggers a best-case scenario.

(b) (2 points) What is the worst case runtime, in terms of n, of this code snippet? Describe what triggers a worst-case scenario.

(c) (4 points) Now analyze the **expected** runtime. Clearly define your random variable. *Hint: Break your random variable into multiple random variables, one per outer loop iteration.*