CMPS 6610 Algorithms – Fall 18

9/17/18

3. Homework

Due **9/24/18** at the beginning of class

1. **Case 3 (8 points)**
   (a) (4 points) Let **case 3'** be a variation of case 3 of the Master theorem, which only requires that $f(n) \in \Omega(n^{\log_b a+\varepsilon})$, but it does not require the regularity condition. What does case 3' imply for $T(n)$? Please provide a proof.
   (b) (4 points) Show that the regularity condition $af(n/b) < cf(n)$ for large enough n and $c < 1$ in case 3 of the Master theorem implies that $f(n) \in \Omega(n^{\log_b a+\varepsilon})$ for some $\varepsilon > 0$.

2. **Recurrences (14 points)**
   Find an asymptotic solution **for as many recurrences below as possible**, either using the Master theorem or by generating a good guess using the recursion tree method for example (no induction required). If the Master theorem does not apply, specify why. Assume that $T(n)$ is constant for sufficiently small n. Justify your answers. *(Note: I can solve all but one.)*

   (a) $T(n) = 8T(\frac{n}{2}) + n^2 \log n$
   (b) $T(n) = 9T(\frac{n}{3}) + n^2 \log n$
   (c) $T(n) = \sqrt{2}T(\frac{n}{2}) + \log n$
   (d) $T(n) = T(\sqrt{n}) + 1$
   (e) $T(n) = 4T(n/3) + n(2 - \cos n)$
   (f) $T(n) = T(\frac{n}{2}) + n(2 - \cos n)$
   (g) $T(n) = T(\frac{n}{2}) + 2^n$