11/28/18

10. Homework

Due **12/5/18** at the beginning of class

Justify all your answers.

1. **To Be or Not to Be ... in P, NP, or Co-NP (4 points)**

   Specify for each of the problems below whether they are in P, NP, and/or *co*–NP.

   (a) Given a directed graph $G = (V, E)$. Is G a DAG?

   (b) Given an undirected graph $G = (V, E)$, and $k > 0$. Is there a subset $S \subseteq V$ with $|S| \leq k$ such that every vertex not in S is adjacent to a vertex in S?

   (c) Given a positive integer a, is a a prime number (i.e., a has no positive integer factors other than 1 and a)?

   (d) Given a directed graph $G = (V, E)$ with non-negative edge weights, and two vertices $s, t \in V$. Compute a shortest path from s to t in G.

2. NP**-completeness (8 points)**

   (a) The **Longest Path** problem takes an undirected weighted graph $G = (V, E)$ with positive edge weights as well as a positive integer k as input, and asks whether there is a simple path of weight at least k in G.
   Show that **Longest Path** is NP-complete.

   (b) The **Subgraph Isomorphism** problem takes two graphs G_1 and G_2 as input and asks whether G_1 is isomorphic to a subgraph of G_2.

   - $G = (V, E)$ is a *subgraph* of $G' = (V', E')$ if $V \subseteq V'$ and $E \subseteq E'$.
   - Two graphs $G = (V, E)$ and $G' = (V', E')$ are *isomorphic* if there exists a bijective map $F : V \to V'$ such that $(u, v) \in E \Leftrightarrow (f(u), f(v)) \in E'$.

   Show that **Subgraph Isomorphism** is NP-complete.

3. $\Pi_1 \leq \Pi_2$ **(10 points)**
   Let Π_1 and Π_2 be decision problems and suppose Π_1 is polynomial-time reducible to Π_2, so, $\Pi_1 \leq \Pi_2$. Answer each of the questions below:

   (a) If $\Pi_2 \in P$ does this imply that $\Pi_1 \in P$?

   (b) If $\Pi_1 \in NP$, does this imply that $\Pi_2 \in NP$?

   (c) If $\Pi_2 \in$ *co*–NP, does this imply that $\Pi_1 \in$ *co*–NP?

   (d) If $\Pi_1 \in NP$, does this imply that Π_2 is NP-complete?

   (e) If $\Pi_1 \notin P$ does this imply that $\Pi_2 \notin P$?

   (f) If Π_2 is NP-complete, does this imply that $\Pi_1 \in NP$?

   (g) If Π_1 is NP-complete, does this imply that $\Pi_2 \in NP$?

   (h) If $\Pi_1 \in NP$ and $\Pi_2 \in P$, what does this imply?

   (i) If Π_1 is NP-complete and $\Pi_2 \in P$, what does this imply?

   (j) If Π_1 is NP-complete and $\Pi_2 \in$ *co*–NP, what does this imply?