11/29/16

9. Homework

Due **12/6/16** at the beginning of class

1. **To be or not to be ... in P, NP, or *co*–NP (6 points)**
   Specify for each of the problems below whether they are in P, NP, and/or *co*–NP. Justify your answers.

   (a) Compute a heap from an array A of n numbers.

   (b) Given an undirected graph $G = (V, E)$, and a number k. Is there a subset $S \subseteq V$ such that every vertex not in S is adjacent to a vertex in S?

   (c) Given an array A of n numbers, and a number k. Does A contain the number k?

   (d) Given an array A of n numbers, and a number k. Is it true that for each subset $S \subseteq A$, the sum of numbers in S does not equal k?

2. **NP-completeness (8 points)**

   (a) The **2-TSP** problem takes an undirected graph $G = (V, E)$ with positive edge weights as well as a positive integer k as input, and asks whether there are **two** closed tours in G such that both tours together visit every vertex in V exactly once, and the total sum of all edge weights on both tours is at most k. Prove that **2-TSP** is NP-complete.

   (b) The **Subgraph Isomorphism** problem takes two graphs G_1 and G_2 as input and asks whether G_1 is isomorphic to a subgraph of G_2.

   - $G = (V, E)$ is a *subgraph* of $G' = (V', E')$ if $V \subseteq V'$ and $E \subseteq E'$.
   - Two graphs $G = (V, E)$ and $G' = (V', E')$ are *isomorphic* if there exists a bijective map $F : V \to V'$ such that $(u, v) \in E \Leftrightarrow (f(u), f(v)) \in E'$.

   Show that **Subgraph Isomorphism** is NP-complete.

3. **NPC and *co*–NP (4 points)**
   Let NPC be the class of NP-complete problems.

   Show that $NPC \cap co\text{–}NP = \emptyset$, under the assumption that $NP \neq co\text{–}NP$.

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4. $\Pi_1 \leq \Pi_2$ **(12 points)**
Let Π_1 and Π_2 be decision problems and suppose Π_1 is polynomial-time reducible to Π_2, so, $\Pi_1 \leq \Pi_2$. Answer and justify each of the questions below:

(a) If $\Pi_2 \in P$ does this imply that $\Pi_1 \in P$?

(b) If $\Pi_1 \in P$ does this imply that $\Pi_2 \in P$?

(c) If $\Pi_1 \in NP$, does this imply that $\Pi_2 \in NP$?

(d) If $\Pi_2 \in$ *co–NP*, does this imply that $\Pi_1 \in$ *co–NP*?

(e) If $\Pi_1 \in NP$, does this imply that Π_2 is NP-complete?

(f) If $\Pi_1 \notin P$ does this imply that $\Pi_2 \notin P$?

(g) If $\Pi_2 \notin P$ does this imply that $\Pi_1 \notin P$?

(h) If Π_2 is NP-complete, does this imply that $\Pi_1 \in NP$?

(i) If Π_1 is NP-complete, does this imply that $\Pi_2 \in NP$?

(j) If Π_1 and Π_2 are NP-complete, is Π_2 polynomially reducible to Π_1?

(k) If $\Pi_1 \in NP$ and $\Pi_2 \in P$, what does this imply?

(l) If Π_1 is NP-complete and $\Pi_2 \in P$, what does this imply?