CMPS 6610 Algorithms – Fall 16

11/1/16

6. Homework

Due **11/8/16** at the beginning of class

1. **Marked Root (1 point)**
   Describe what sequence of operations in a Fibonacci heap would result in a root that is marked.

2. **Fibonacci Heap Path (6 points)**

   (a) (4 points) For any $n \geq 1$, describe how to form a sequence of Fibonacci heap operations that creates a Fibonacci heap in which all n nodes form a single path of height n.

   (b) (2 points) What happens to your constructed Fibonacci heap if you perform a single DECREASE_KEY operation on the node with the maximum key (i.e., the node furthest from the root) in such a way that the new value causes a violation of the heap property?

3. **Second_Smallest (5 points)**

   (a) (2 points) How fast can you compute the second smallest element in a Fibonacci heap? Justify the correctness and runtime of your answer.

   (b) (3 points) Modify the Fibonacci heap data structure to implement a procedure computing the second smallest element in constant time.

4. **MST With Distinct Edge Weights (6 points)**
   Let $G = (V, E; w)$ be an edge-weighted, undirected connected graph where the edge weights are all distinct.

   (a) (3 points) Show that the MST of G is unique.

   (b) (3 points) Show that the second-best MST of G is not unique.

5. **Adding and Deleting Edges in an MST (8 points)**

   Let $G = (V, E; w)$ be an edge-weighted, undirected connected graph, and let T be an MST for G.

   (a) (4 points) Assume that a new edge e is added between two existing vertices. Describe how to find an MST of the new graph in time proportional to $|V|$.

   (b) (4 points) Now assume that an edge is deleted from G and assume that this operation does not disconnect G. Describe how to find an MST of the new graph in time proportional to $|E|$. Justify the correctness of your algorithm.