

## 6. Homework

Due **11/8/16** at the beginning of class

**1. Marked Root (1 point)**

Describe what sequence of operations in a Fibonacci heap would result in a root that is marked.

**2. Fibonacci Heap Path (4 points)**

For any  $n \geq 1$ , describe how to form a sequence of Fibonacci heap operations that creates a Fibonacci heap in which all  $n$  nodes form a single path of height  $n$ .

**3. Second Smallest (5 points)**

- (a) (2 points) How fast can you compute the second smallest element in a Fibonacci heap? Justify the correctness and runtime of your answer.
- (b) (3 points) Modify the Fibonacci heap data structure to implement a procedure computing the second smallest element in constant time.

**4. MST With Distinct Edge Weights (6 points)**

Let  $G = (V, E; w)$  be an edge-weighted, undirected connected graph where the edge weights are all distinct.

- (a) (3 points) Show that the MST of  $G$  is unique.
- (b) (3 points) Show that the second-best MST of  $G$  is not unique.

**5. Adding Edges in an MST (4 points)**

Let  $G = (V, E; w)$  be an edge-weighted, undirected connected graph, and let  $T$  be an MST for  $G$ .

Now assume that a new edge  $e$  is added between two existing vertices. Describe how to find an MST of the new graph in time proportional to  $|V|$ .