CMPS 6610 Introduction to Algorithms – Fall 16

9/13/16

2. Homework

Due **9/22/16** at the beginning of class

1. **BST Generator (6 points)**
   Describe an efficient algorithm that generates a perfectly balanced binary search tree of height h with keys 0 through $2^{h+1} - 2$. Your algorithm should be described in words and in pseudocode. Analyze the running time.

2. **Heaps (6 points)**
   Let H be a max-heap storing n keys. Give an efficient algorithm for printing all keys in H that are greater than or equal to a given query key q. Your algorithm should run in $O(k)$ time, where k is the number of keys printed. Justify that your algorithm has this runtime.

3. **Bitonic (6 points)**
   Let $A[0..n-1]$ be an array containing a *bitonic* sequence of n distinct numbers. A sequence is called *bitonic* if it consists of a decreasing sequence followed by an increasing sequence. For example $8, 5, 3, -1, -4, 2, 6$ is bitonic, while $8, 5, 3, 2, 1, 7, 6$ is not.

   Describe a divide-and-conquer algorithm that finds the minimum element in a bitonic array in $O(\log n)$ time.

4. **Guessing and Induction (10 points)**

   For the recurrences below, use the recursion tree method to find a good guess of what they could solve to asymptotically (make your guess as tight as possible). Then prove that $T(n)$ is in big-Oh of your guess by big-Oh induction; you do not have to prove the base case.

   Every recurrence below is stated for $n \geq 2$, and the base case is $T(1) = 1$.

   (a) (5 points) $T(n) = 4T(n/2) + 3n$

   (b) (5 points) $T(n) = 5T(n/3) + n^4$