CMPS 3130/6130 Computational Geometry – Spring 17

4/6/17

7. Homework

Due **Thursday 4/13/17** before class.

1. **Range Counting (10 points)**
   (a) Describe how to augment a 1D range tree of n elements such that range **counting** queries can be answered in $O(\log n)$ time. Argue that your augmentation does not change the asympotic preprocessing time and the asymptotic space complexity.
   (b) Describe how to augment a 2D range tree of n elements such that range **counting** queries can be answered in $O(\log^2 n)$ time.

2. **Sign Vectors (10 points)**
   Consider an arrangement $\mathcal{A}$ of six lines $l_1, l_2, l_3, l_4, l_5, l_6$, and let f be an arbitrary vertex, edge, or face of $\mathcal{A}$. Then f has an associated *sign vector* $(s_1, s_2, s_3, s_4, s_5, s_6)$, where for each $1 \leq i \leq 6$:

$$s_i = \begin{cases} +1, & \text{if } f \text{ lies above } l_i \\ 0, & \text{if } f \text{ lies on } l_i \\ -1, & \text{if } f \text{ lies below } l_i \end{cases}$$

   (a) For each of the sign vectors below, give an arrangement of six lines that has a vertex, edge, or face with this sign vector. Label the lines and mark the vertex, edge, or face. Make the arrangement simple, if possible, or argue why the arrangement cannot be simple.
   i. $(+1, +1, +1, +1, +1, +1)$ ii. $(+1, 0, 0, -1, -1, -1)$ iii. $(-1, 0, 0, -1, +1, -1)$ iv. $(+1, -1, -1, -1, -1, -1)$
   (b) Can one find a single arrangement of lines that contains a vertex, edge, or face for each of the four sign vectors? Argue why or why not.

3. **Fractional Cascading (10 points)**
   Give pseudocode for a two-dimensional range query in a layered range tree that uses fractional cascading. You only need to consider the left side of the query by x-range after the split node, i.e., you only need to modify the code on slide 11 of the range searching slides. Make sure to describe the pointer attributes in the tree nodes that you are using in your code.