CMPS 3130/6130 Computational Geometry – Spring 17

3/23/17

6. Homework

Due **Thursday 4/6/17** before class.

1. **Worst-Case DT Runtime (7 points)**
   The randomized incremental construction of the Delaunay triangulation of a set of n points in the plane takes $\Omega(n^2)$ time in the worst-case. That means that, for each n, there is a set of n points together with a particular input order such that the algorithm executes $\Omega(n^2)$ edge flips.

   What properties are required to cause that many flips? Please sketch the construction of such a bad input (it should work for general n). *(Hint: It might help to play with one of the Delaunay triangulation programs. You can download Voroglide and run it with appletviewer from the command line.)*

2. **Railway Tracks (9 points)**
   On n parallel railway tracks n trains are going with constant speeds $v_1, \ldots, v_n$. At time $t = 0$ the trains are at positions $k_1, \ldots, k_n$.

   Give an $O(n \log n)$ time algorithm that detects all trains that at some moment in time are leading.
   (Hint: Use halfplane intersection.)

3. **Dual triangle (6 points)**
   Consider a (solid) triangle Δpqr with corner points p, q, r. Describe its dual.

4. **Linear Separator (8 points)**
   Let $R = \{r_1, \ldots, r_m\}$ be set of m red points, and let $B = \{b_1, \ldots, b_n\}$ be a set of n blue points in the plane. A line l is called a **linear separator** if all points of R lie on one side of l and all points of B lie on the other side. (You may assume appropriate general position, and may disregard points that lie exactly on the line.)

   Use point-line duality to develop an algorithm for this problem which runs in expected linear time. *(Hint: Linear Programming.)*