CMPS 3130/6130 Computational Geometry – Spring 17

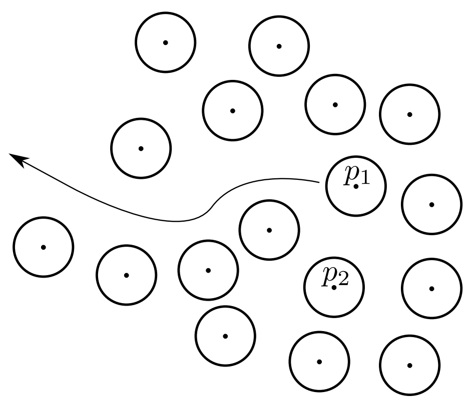
3/7/17

5. Homework

Due **Tuesday 3/14/17** before class.

1. **Parabolic Arc (5 points)**
   Give an example where the parabola defined by some site p_i contributes more than one arc to the beach line. Can you give an example where it contributes a linear number of arcs?

2. **Disk Escape (10 points)**
   Assume you are given a set of n non-intersecting unit disks in the plane with center points $p_1, \ldots, p_n$. Determine whether it is possible for the disk with center point p_1 to *escape* from the others. The disk can escape iff it can continuously move arbitrary far away from the other disks without shifting or intersecting the other disks. In the example below, the disk with center point p_1 can escape, but the disk with center point p_2 cannot.

   Give an $O(n \log n)$-time algorithm to solve this problem. Your algorithm should use a Voronoi diagram. Either indicate that the disk cannot escape, or output a path along which to move the center point of the disk.

3. **Sum of Edge Lengths (5 points)**
   It appears that illegal edges are often long edges, so it is a natural question to ask whether the Delaunay triangulation might minimize edge lengths. Give an example which shows that the Delaunay triangulation of a point set is not always the triangulation with the minimum sum of edge lengths.

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4. **Gabriel Graph (10 points)**
   Let P be a set of n points in the plane. The *Gabriel graph* $GG(P)$ is defined as follows: Two points $p, q \in P$ are connected by an edge in $GG(P)$ iff the circle with diameter pq does not contain any other point of P in its interior.

   (a) Prove that $DT(P)$ contains $GG(P)$. I.e., every edge in $GG(P)$ is also a Delaunay edge.

   (b) Prove that p and q are adjacent in $GG(P)$ if and only if the Delaunay edge between p and q intersects its dual Voronoi edge.

5. **Extra Credit: Reverse Voronoi (10 extra credit points)**
   Suppose we are given a subdivision of the plane into n convex regions. We suspect that this subdivision is a Voronoi diagram, but we do not know the sites. Develop an algorithm that finds a set of n point sites whose Voronoi diagram is exactly the given subdivision, if such a set exists.