CMPS 3130/6130 Computational Geometry – Spring 17

1/26/17

2. Homework

Due **2/2/17** before class

1. **Lower Bounds (8 points)**
   Consider the following problems:

   SORTING: Given a set $X = \{x_1, \ldots, x_n\}$ of n numbers, output the same numbers in non-decreasing order.

   ELEMENT UNIQUENESS: Given a set $X = \{x_1, \ldots, x_n\}$ of n numbers, are there i, j, with $i \neq j$, such that $x_i = x_j$?

   CLOSEST PAIR: Given a point set $P = \{p_1, \ldots, p_n\} \in \mathbb{R}^2$, output the closest pair of points in P.

   ALL NEAREST NEIGHBORS: Given a point set $P = \{p_1, \ldots, p_n\} \in \mathbb{R}^2$. Compute for each point in P its *nearest neighbor* in P (i.e., point at minimum distance).

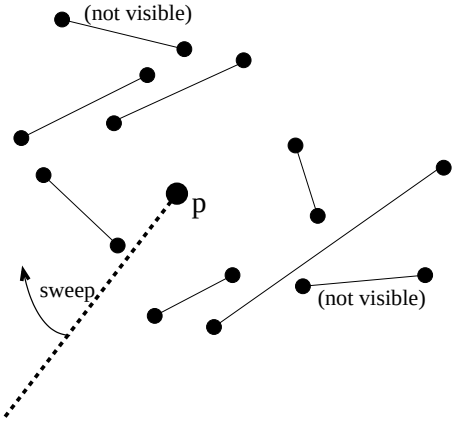
   (a) Prove a lower bound of $\Omega(n \log n)$ for SORTING, by reducing from ELEMENT UNIQUENESS (i.e., by using the knowledge that ELEMENT UNIQUENESS has a lower bound of $\Omega(n \log n)$).

   (b) Prove a lower bound of $\Omega(n \log n)$ for CLOSEST PAIR by reducing from an appropriate problem.

   (c) Prove a lower bound of $\Omega(n \log n)$ for ALL NEAREST NEIGHBORS by reducing from an appropriate problem.

2. **Visible Segments Sweep (8 points)**

   Let S be a set of n disjoint line segments in the plane, and let p be a point not on any of the line segments of S. We say that the point p *sees* a line segment s if there is a point $q \in s$ such that the segment pq does not intersect any other line segment of S. We wish to determine all line segments of S that p can see.
   Give an $O(n \log n)$ time algorithm for this problem that uses a rotating half-line with its endpoint at p.



3. **Guarding Boundary vs. Interior (5 points)**
   Give an example of a polygon together with a placement of vertex guards, such that the whole polygon boundary is guarded but not the whole interior.

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4. **Guarding the Fleur-de-Lis (9 points)**
   For the simple polygon P below:

   (a) Apply the method employed by the 3-coloring-based proof to obtain a set of at most $\lfloor \frac{n}{3} \rfloor$ **vertex guards** that guard P.

   (b) By inspection, obtain the minimum number of **vertex guards** necessary to guard P. Justify your answer.

   (c) By inspection, obtain the minimum number of **point guards** necessary to guard P, i.e., guards are allowed to be anywhere in the interior or on the boundary of P. Justify your answer.