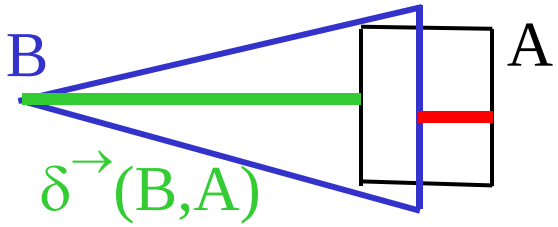
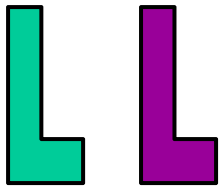
CMPS 3130/6130 Computational Geometry
Spring 2015

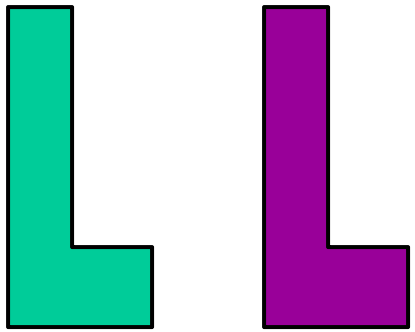


Shape Matching

Carola Wenk

When are two shapes similar?





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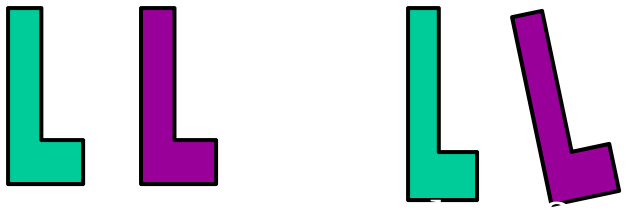

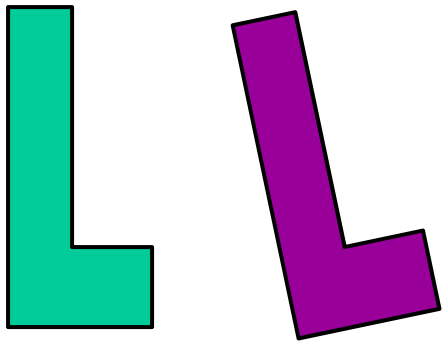

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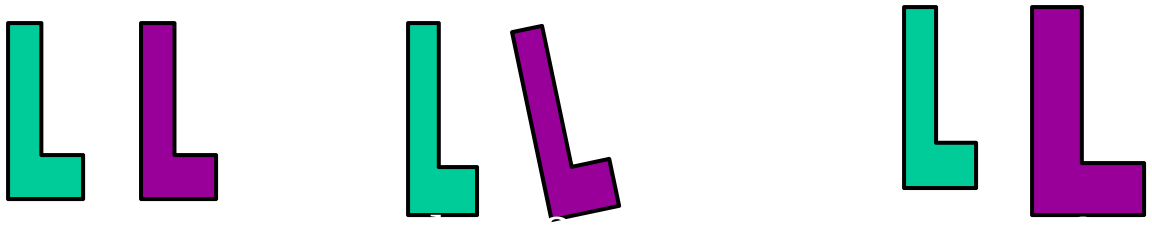

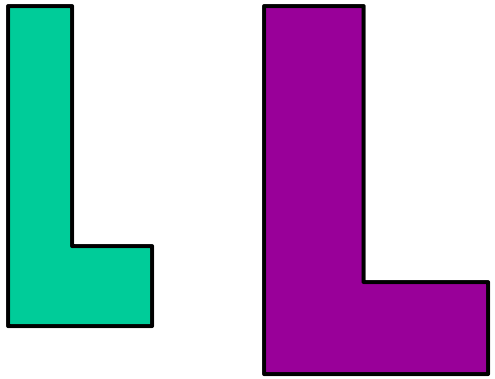

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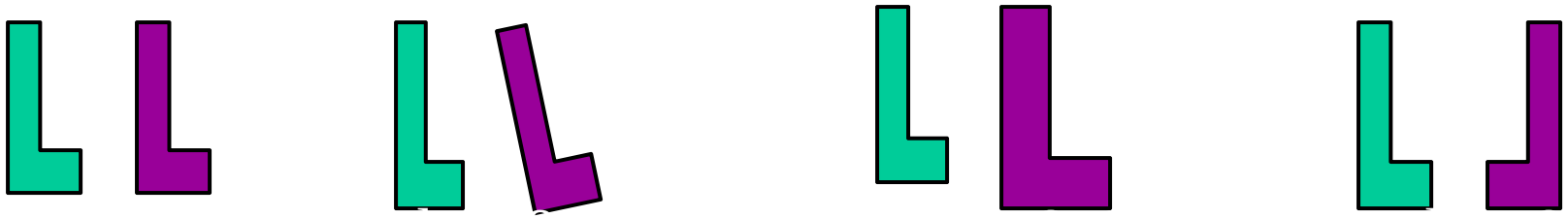

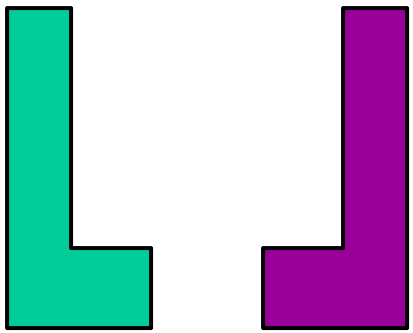

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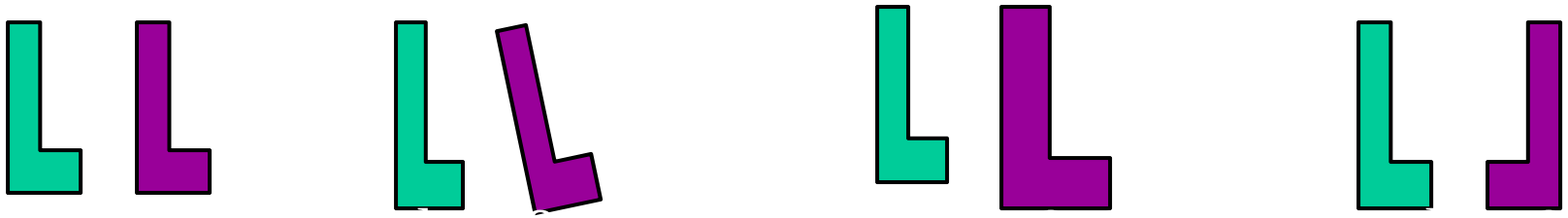

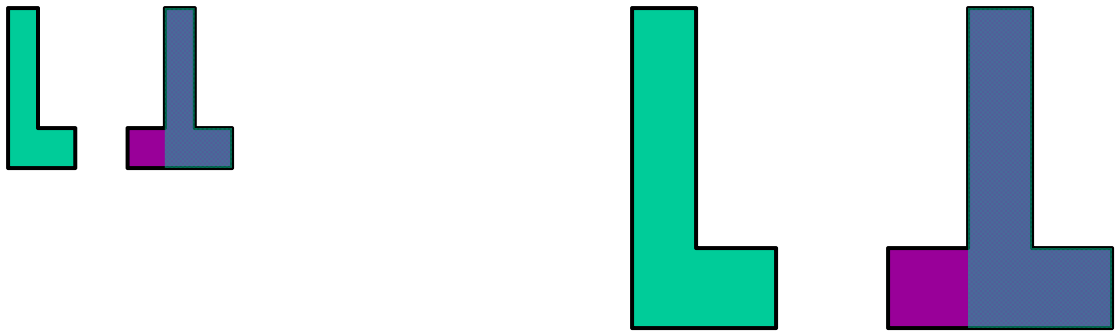

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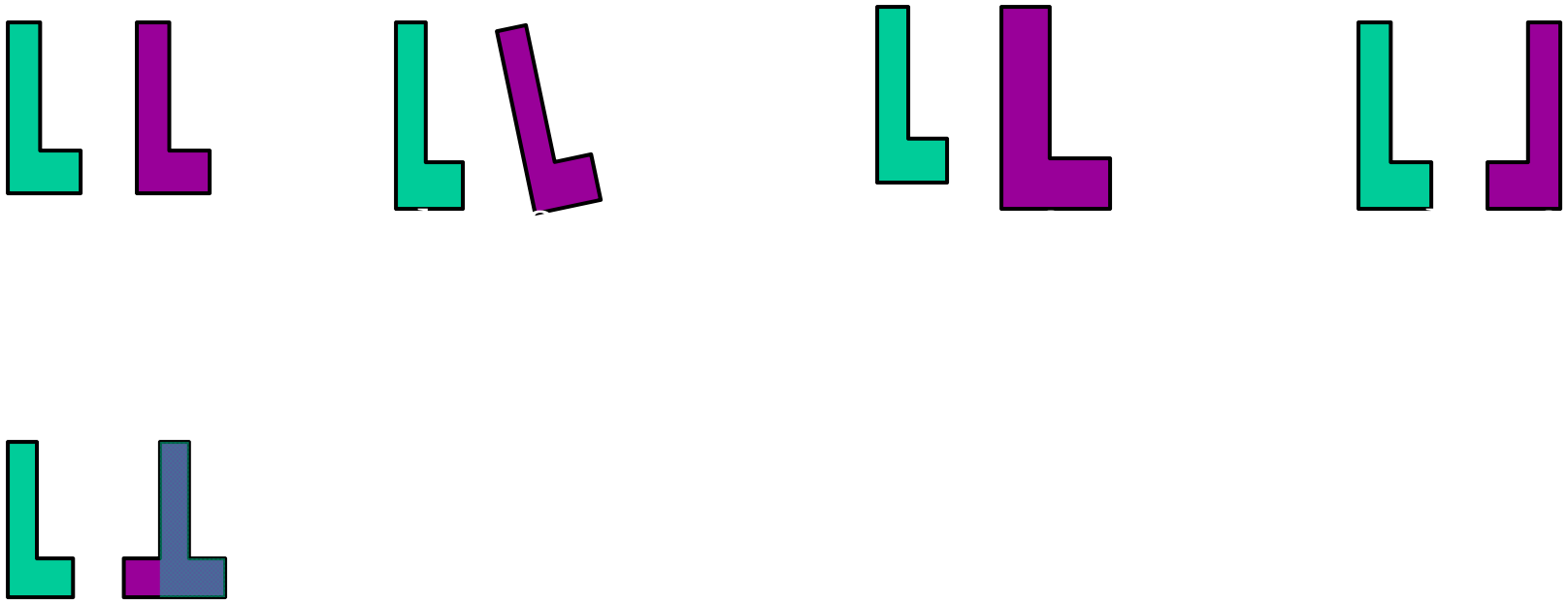
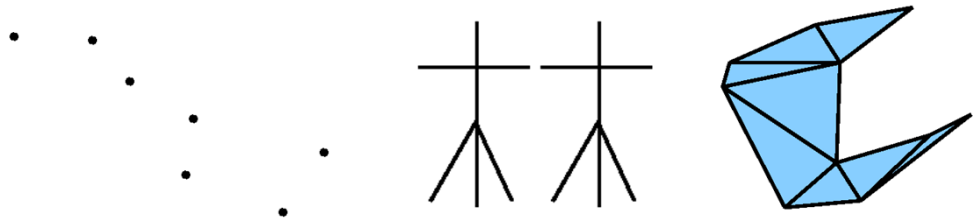


Geometric Shape Matching

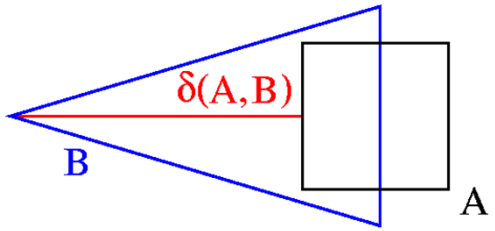
Given:

Two geometric shapes, each composed of a number of basic objects such as



points line segments triangles

An application dependant distance measure δ, e.g., the Hausdorff distance



A set of transformations $\boldsymbol{T}$, e.g., translations, rigid motions, or none

Matching Task: Compute $\min_{T \in \boldsymbol{T}} \delta(T(A),B)$

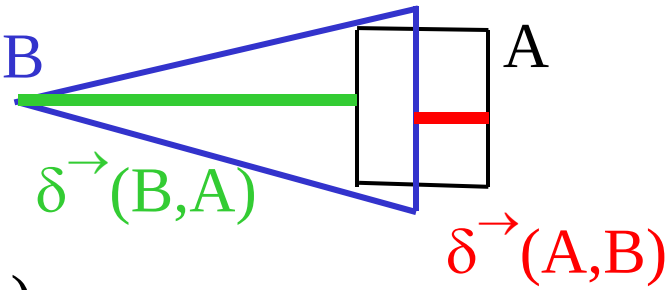
Hausdorff distance

- Directed Hausdorff distance

$$\vec{\delta}(A,B) = \max_{a \in A} \min_{b \in B} \| a-b \|$$

- Undirected Hausdorff-distance

$$\delta(A,B) = \max (\vec{\delta}(A,B) , \vec{\delta}(B,A))$$



- If A, B are discrete point sets, |A|=*m*, |A|=*n*
  - In d dimensions: Compute in O(*mn*) time brute-force
  - In 2 dimensions: Compute in O((*m*+*n*) log (*m*+*n*) time
    - Compute $\vec{\delta}(A,B)$ by computing VD(B) and performing nearest neighbor search for every point in A

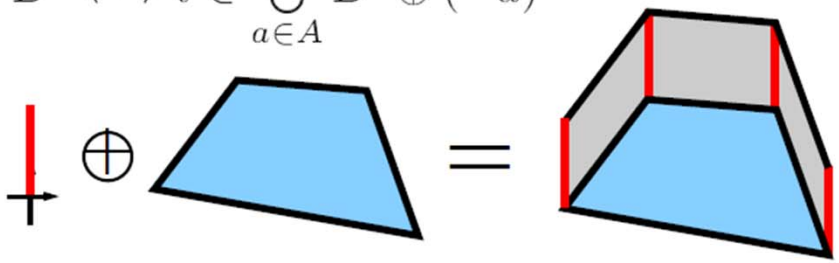
Hausdorff distance under translations

Decision problem, transformation space approach:

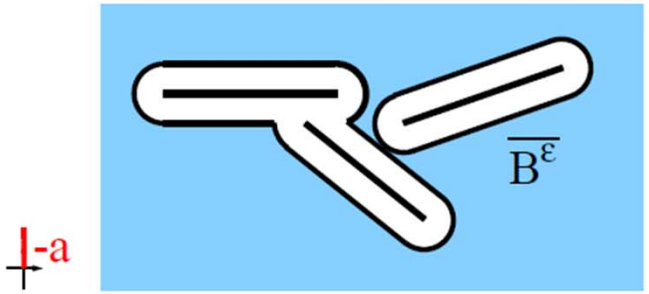
$$\vec{\delta}_H(A+t,B) \leq \epsilon \iff A+t \subseteq B^\epsilon \iff t \in \bigcup_{a \in A} \overline{B^\epsilon} \oplus (-a)$$

- Minkowski sum

  $$a \oplus b = \bigcup_{x \in a} \bigcup_{y \in b} x+y$$



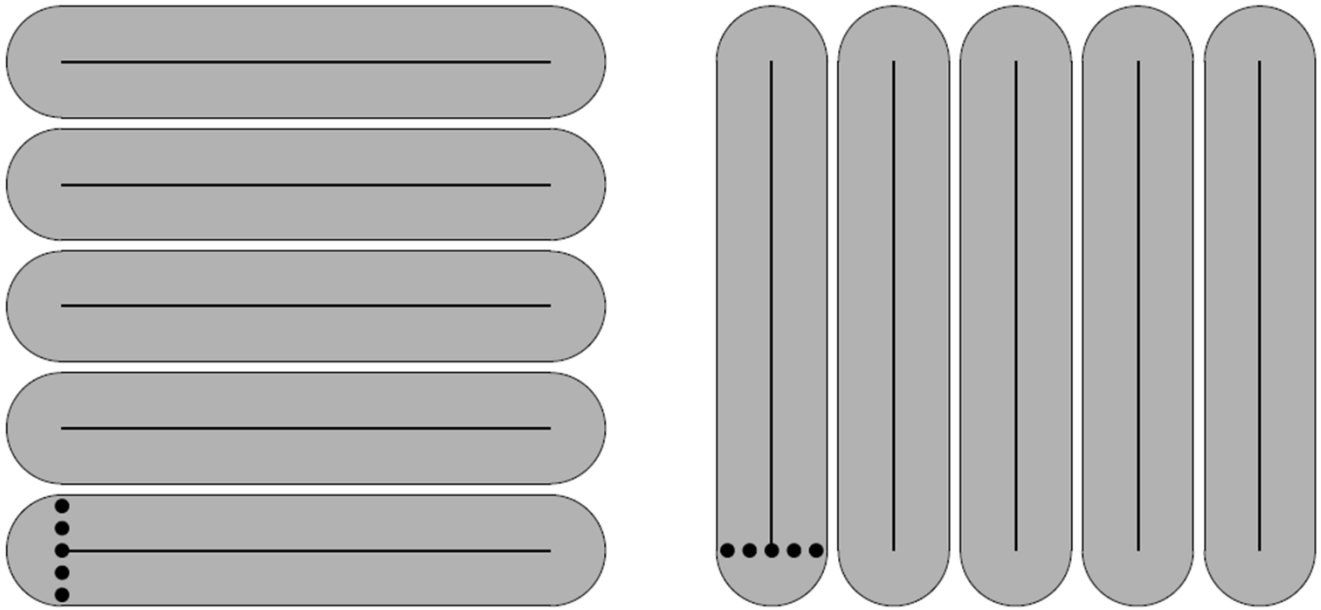
- Arrangement construction



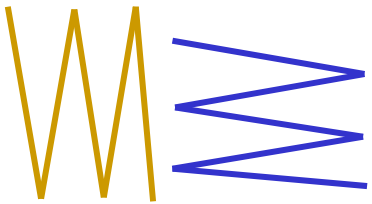
- Segments, $\mathbb{R}^2$: $O((n \cdot n)^2)$ arrangement complexity (AST94)
- Simplices, $\mathbb{R}^d$, L_∞: $O((\text{Complexity}(B^\epsilon))^d n^d) \leq O(n^{d^2+d})$
- Triangles, $\mathbb{R}^3$, L_2 : $O(n^{6+\delta})$ arrangement complexity

Hausdorff distance

• $\Omega(m^2n^2)$ lower bound construction for directed Hausdorff distance of line segments under translations

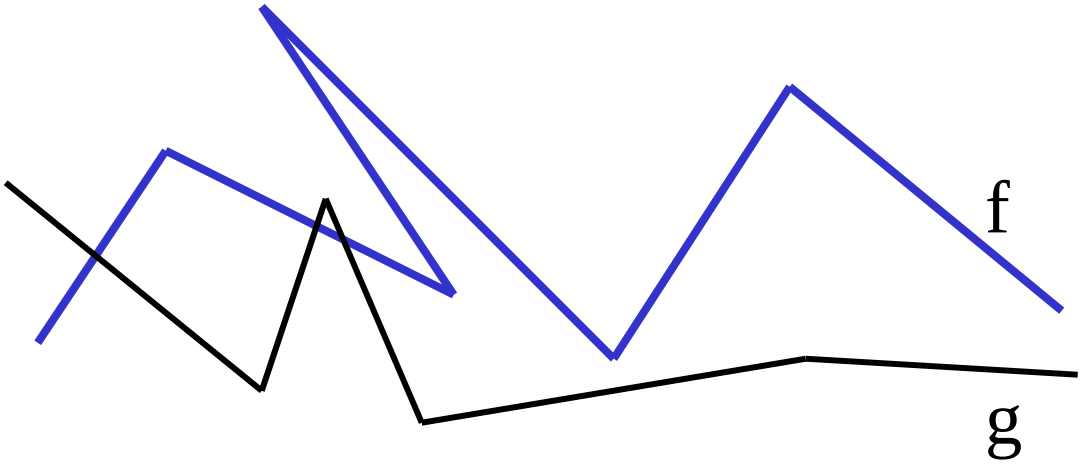


Comparing Curves and Trajectories



Polygonal Curves

- Let $f,g:[0,1] \rightarrow R^d$ be two **polygonal curves** (i.e., piecewise linear curves)

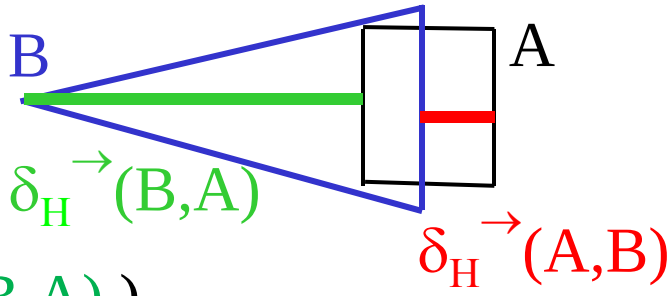


- What are good distance measures for curves?
  - Hausdorff distance?
  - Fréchet distance?

When Are Two Curves „Similar“?

- Directed Hausdorff distance

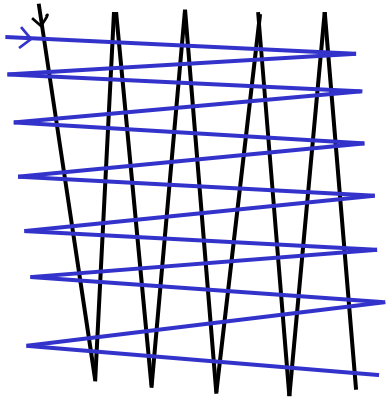
$$\delta_H^{\rightarrow}(A,B) = \max_{a \in A} \min_{b \in B} \| a-b \|$$



- Undirected Hausdorff-distance

$$\delta_H(A,B) = \max(\delta_H^{\rightarrow}(A,B), \delta_H^{\rightarrow}(B,A))$$

But:

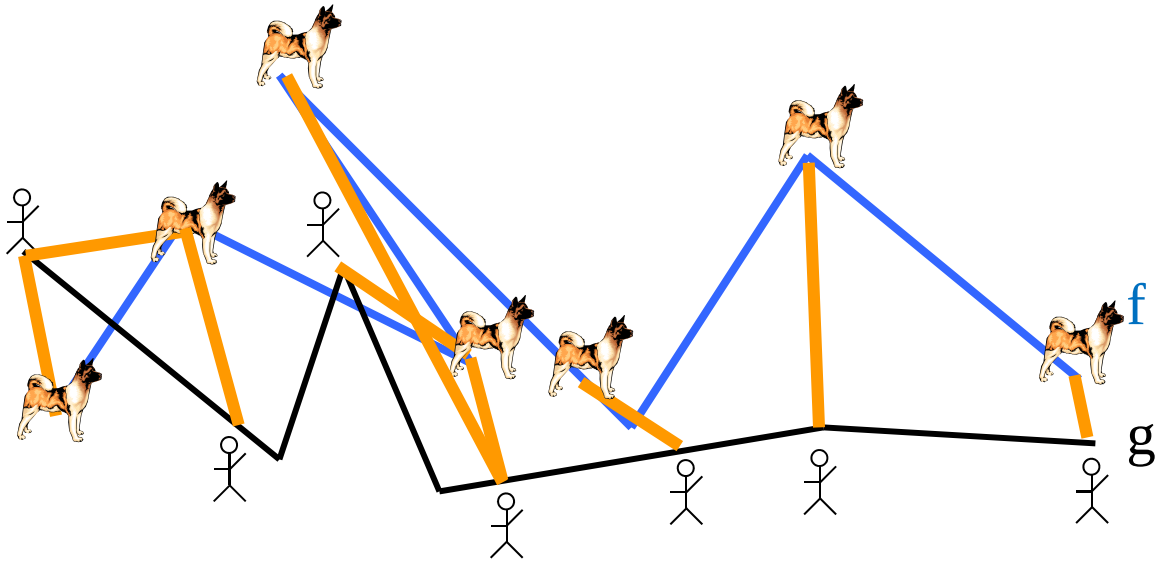


- Small Hausdorff distance
- When considered as curves the distance should be large
- The Fréchet distance takes the continuity of the curves into account

Fréchet Distance for Curves

$$\delta_F(f,g) = \inf_{\alpha,\beta:[0,1]\to[0,1]} \max_{t\in[0,1]} \|f(\alpha(t))-g(\beta(t))\|$$

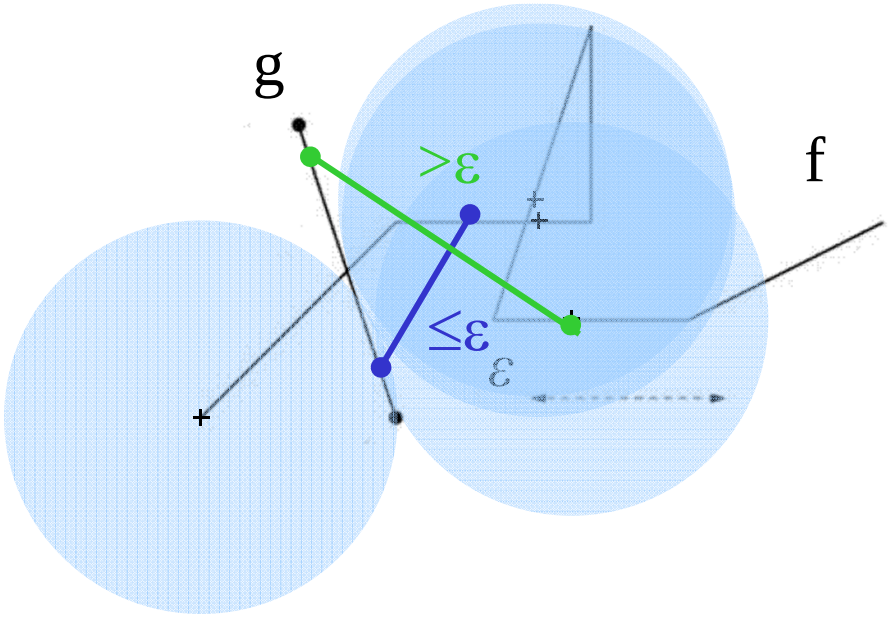
where α and β range over continuous monotone increasing reparameterizations only.

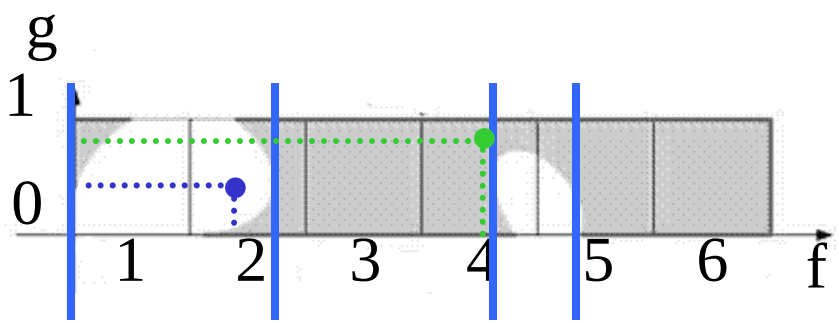


- Man and dog walk on one curve each
- They hold each other at a **leash**
- They are only allowed to go forward
- δ_F is the minimal possible leash length

[F06] M. Fréchet, Sur quelques points de calcul fonctionel, *Rendiconti del Circolo Mathematico di Palermo* 22: 1-74, 1906.

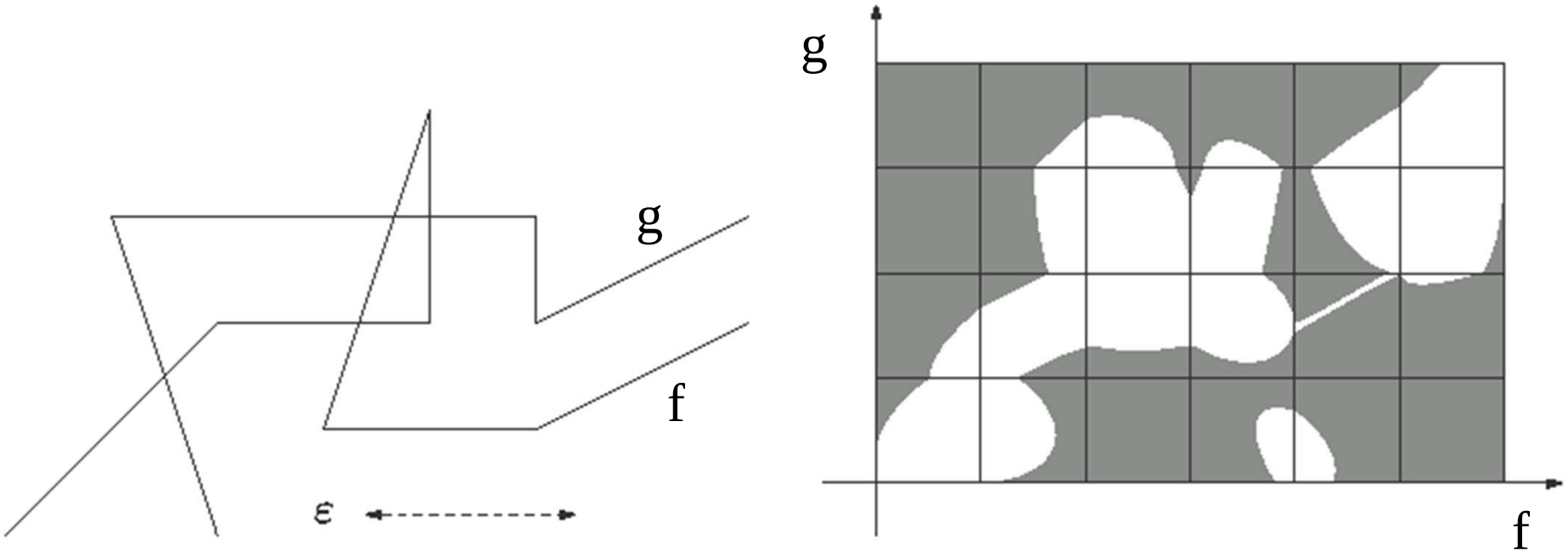
Free Space Diagram





- Let ε > 0 fixed (eventually solve decision problem)
- $F_\varepsilon(f,g) = \{ (s,t) \in [0,1]^2 \mid \| f(s) - g(t) \| \leq \varepsilon \}$ *white points* **free space** of f and g
- The free space in one cell is an ellipse.

Free Space Diagram



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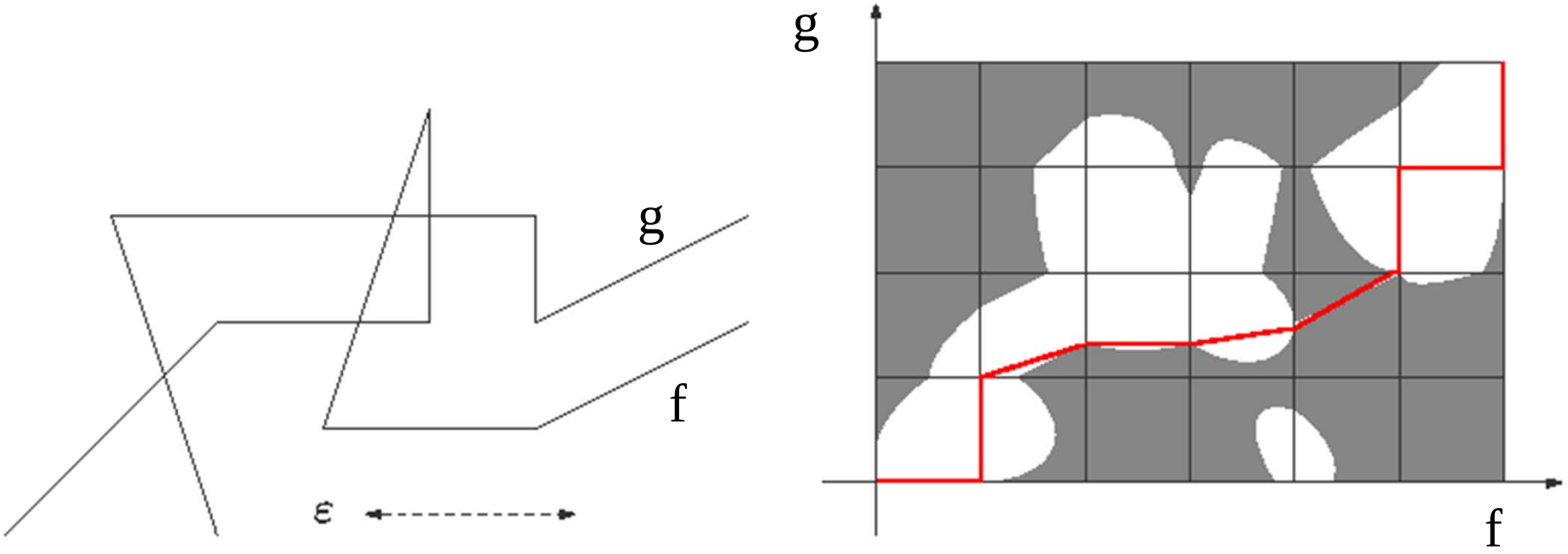
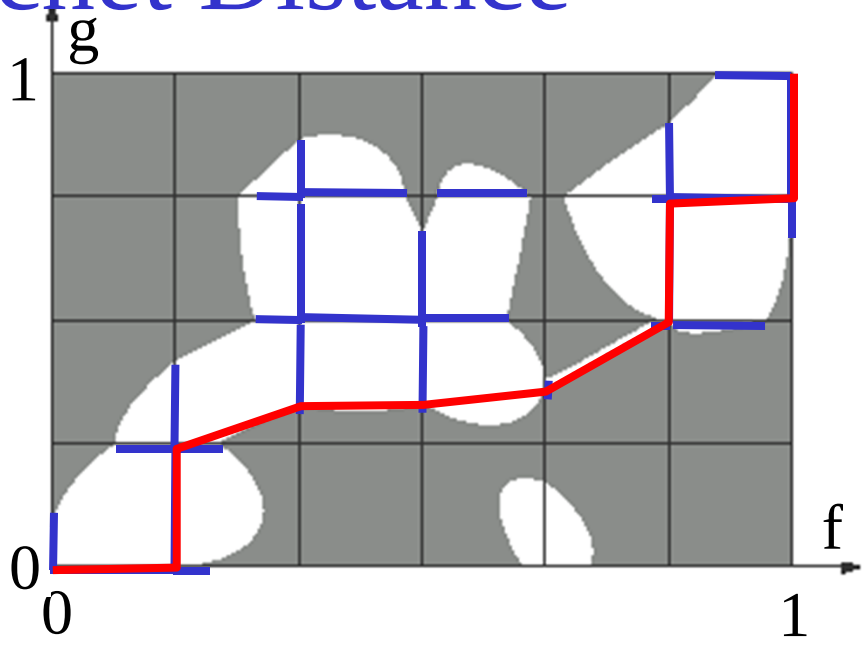


- Monotone path encodes reparametrizations of f and g
- $\delta_F(f,g) \leq \varepsilon$ iff there is a monotone path in the free space from (0,0) to (1,1)

Compute the Fréchet Distance

- **Solve the decision problem**
  $\delta_F(f,g) \leq \varepsilon$ in O(mn) time:
  - Find monotone path using DP:
  - On each cell boundary compute the interval of all points that are reachable by a monotone path from (0,0)
  - Compute a monotone path by backtracking



- **Solve the optimization problem**
  - In practice in O(mn log b) time with binary search and b-bit precision
  - In O(mn log mn) time [AG95] using parametric search (using Cole's sorting trick)
  - In O(mn $\log^2$mn) expected time [CW09] with randomized red/blue intersections

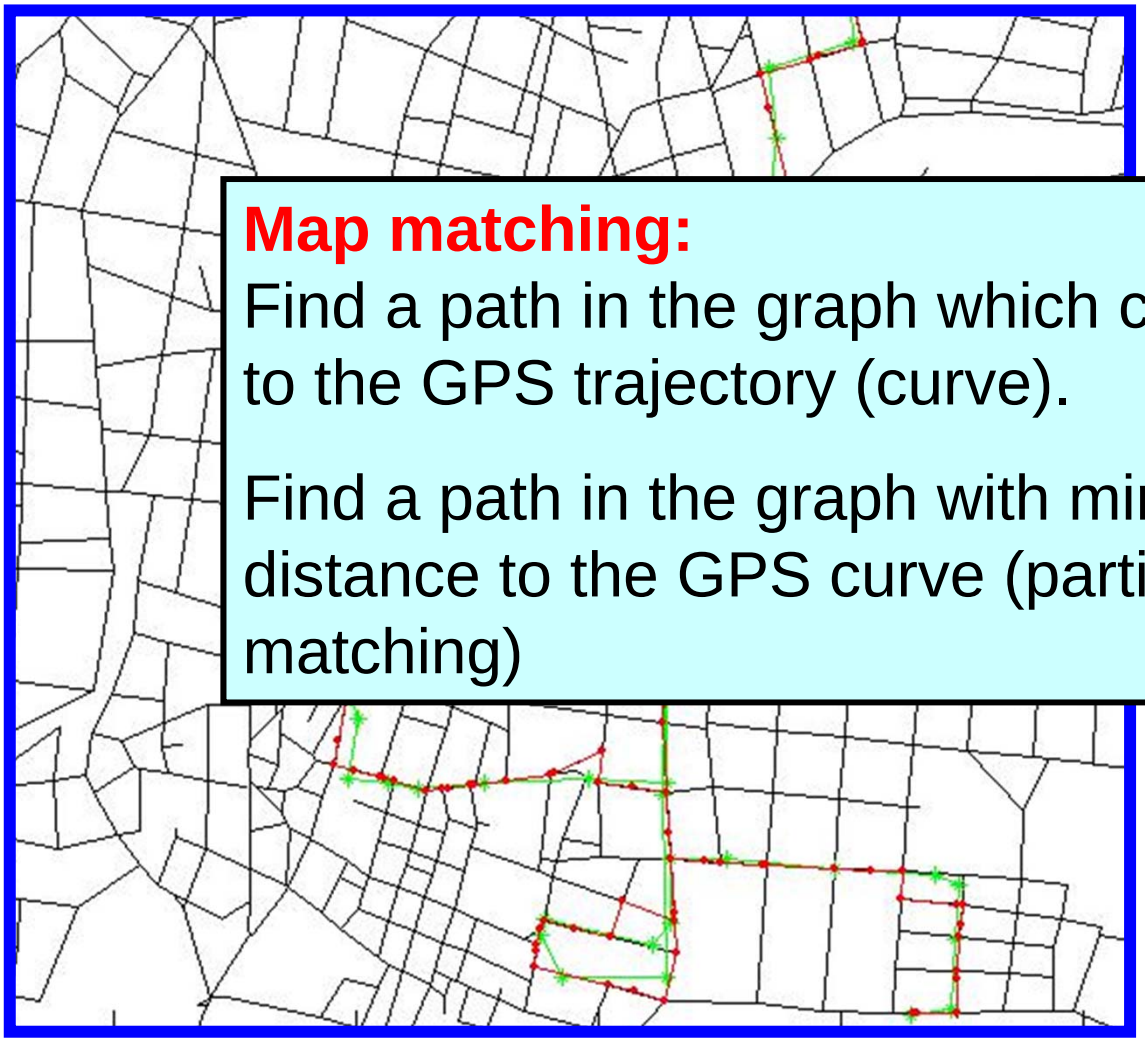
[AG95] H. Alt, M. Godau, Computing the Fréchet distance between two polygonal curves, *IJCGA* 5: 75-91, 1995.
[CW10] A.F. Cook IV, **C. Wenk**, Geodesic Fréchet Distance Inside a Simple Polygon, ACM TALG 7(1), 19 pages, 2010.

GPS Trajectories for Dynamic Routing

- Navigation systems answer shortest path queries based on travel times on road segments
- Usually based on static travel-time weights derived from speed limits
- **Goal:** Develop a navigation system which answers routing queries **based on the current traffic situation**

- **Current traffic situation:**
  - Database of current travel times per road segment
  - Use GPS trajectory data from vehicle fleets

GPS Trajectories from Vehicles



Map matching:
Find a path in the graph which corresponds to the GPS trajectory (curve).

Find a path in the graph with minimal distance to the GPS curve (partial matching)

1) Measurement error: GPS points do not lie d map

error: trajectory oduct and sampled every 30s

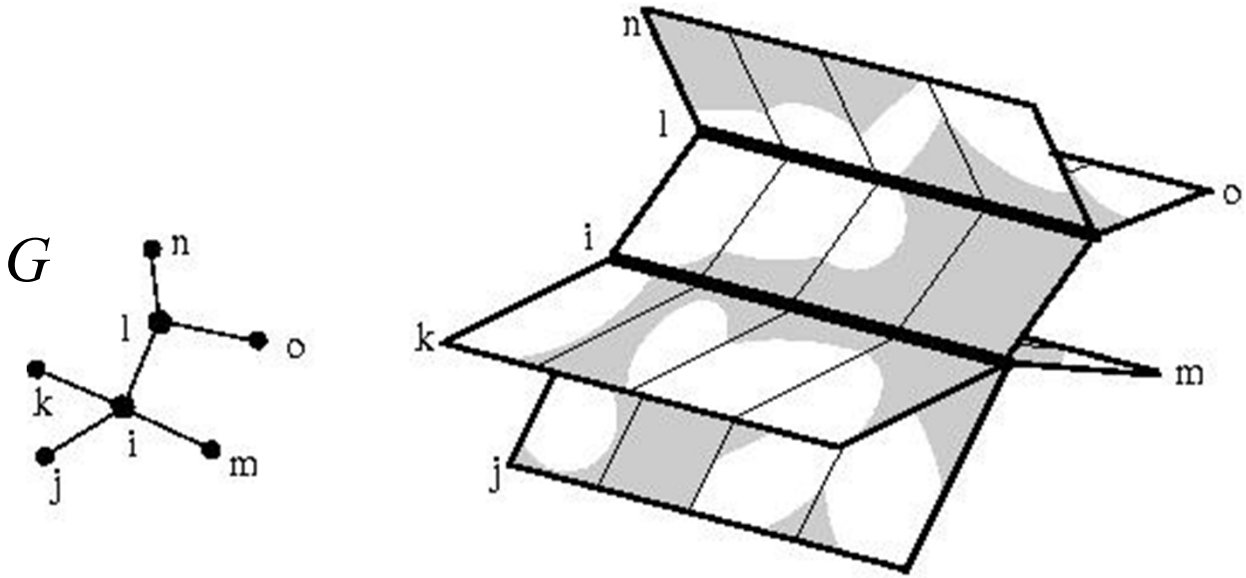
⇒ The GPS trajectory does not lie on the road map

Road map of Athens

GPS trajectory

Corresponding path in the road map

Free Space Surface

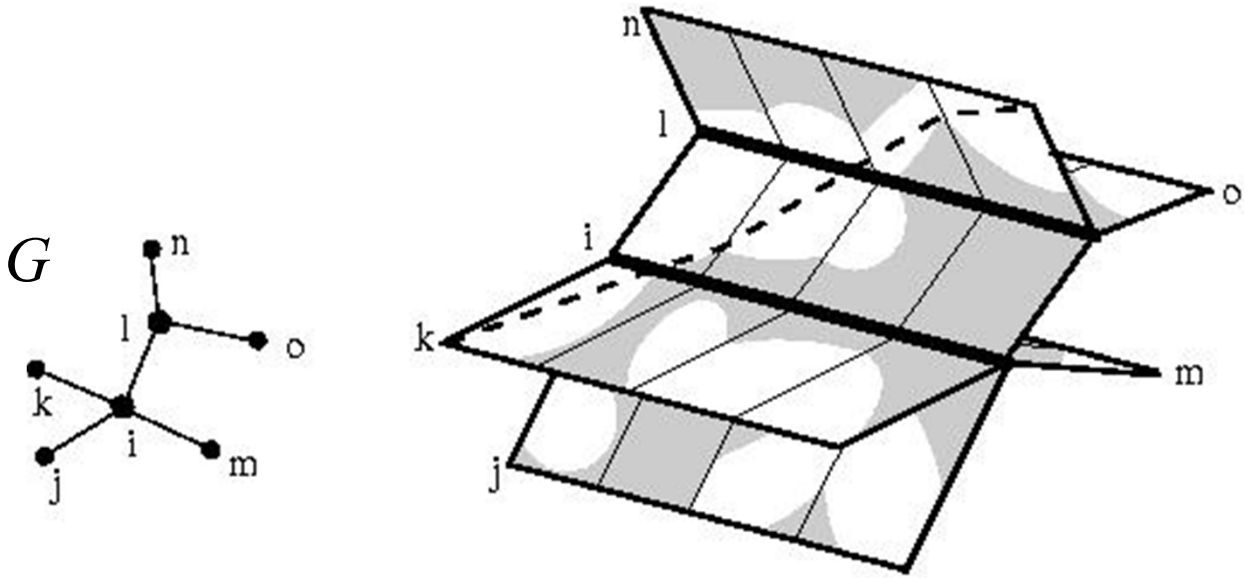


- Glue the free space diagrams $FD_{i,j}$ together according to adjacency information in *G*

→ **Free space surface of f and *G***

[AERW03] H. Alt, A. Efrat, G. Rote, **C. Wenk**, Matching Planar Maps, *J. of Algorithms* 49: 262-283, 2003.
[BPSW05] S. Brakatsoulas, D. Pfoser, R. Salas, **C. Wenk**, On Map-Matching Vehicle Tracking Data , VLDB 853-864 , 2005.
[WSP06] **C. Wenk**, R. Salas, D. Pfoser, Adressing the Need for Map-Matching Speed…, SSDBM: 379-388, 2006.

Free Space Surface



- **Task:** Find a **monotone path** in the free space surface that
  - starts in a lower left corner
  - and ends in an upper right corner

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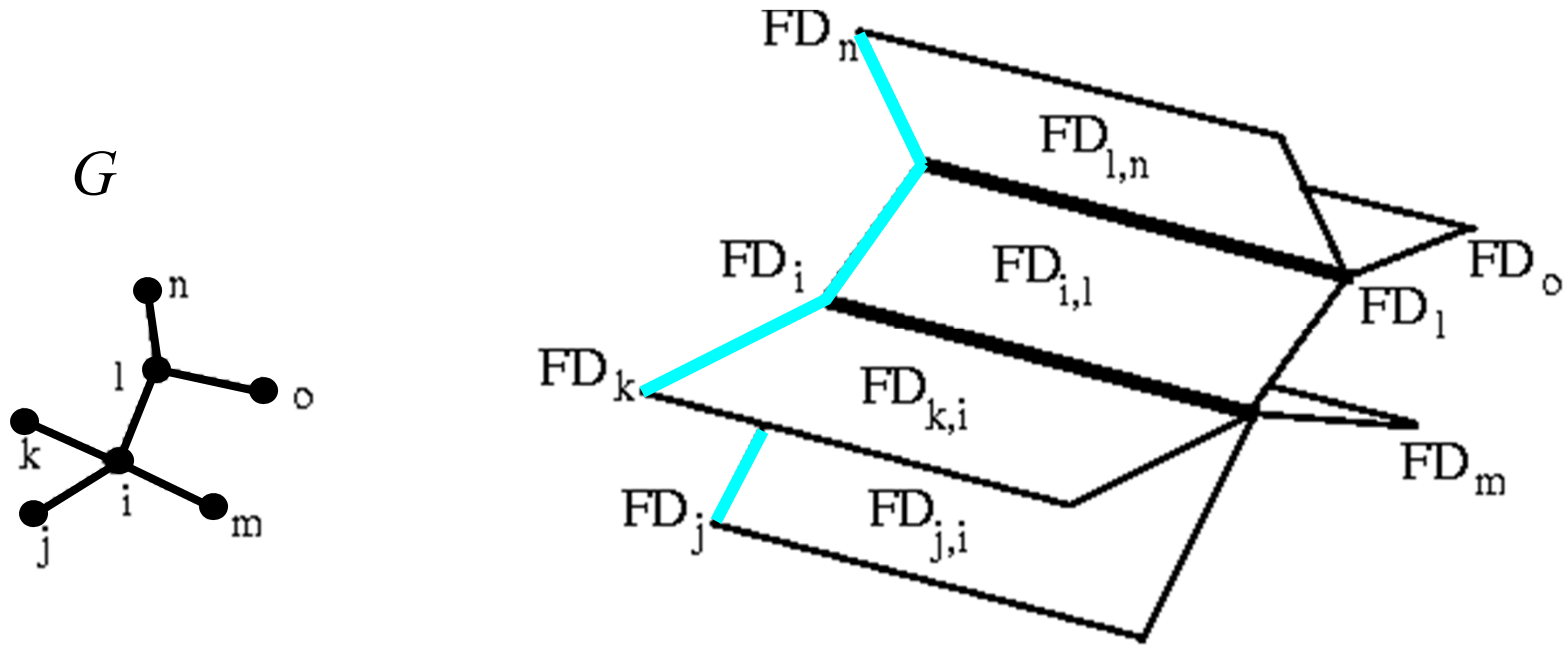
Sweep



- Sweep all $FD_{i,j}$ with a sweep line from left to right

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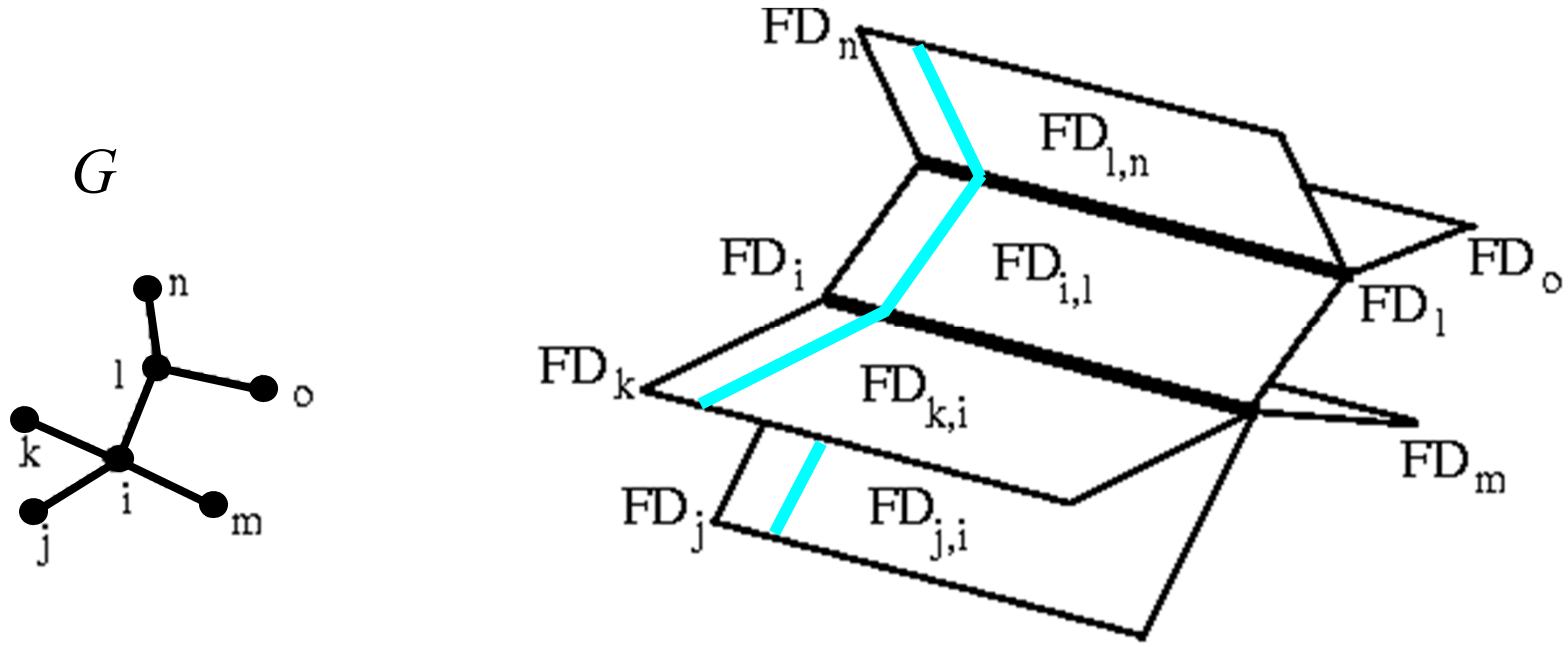

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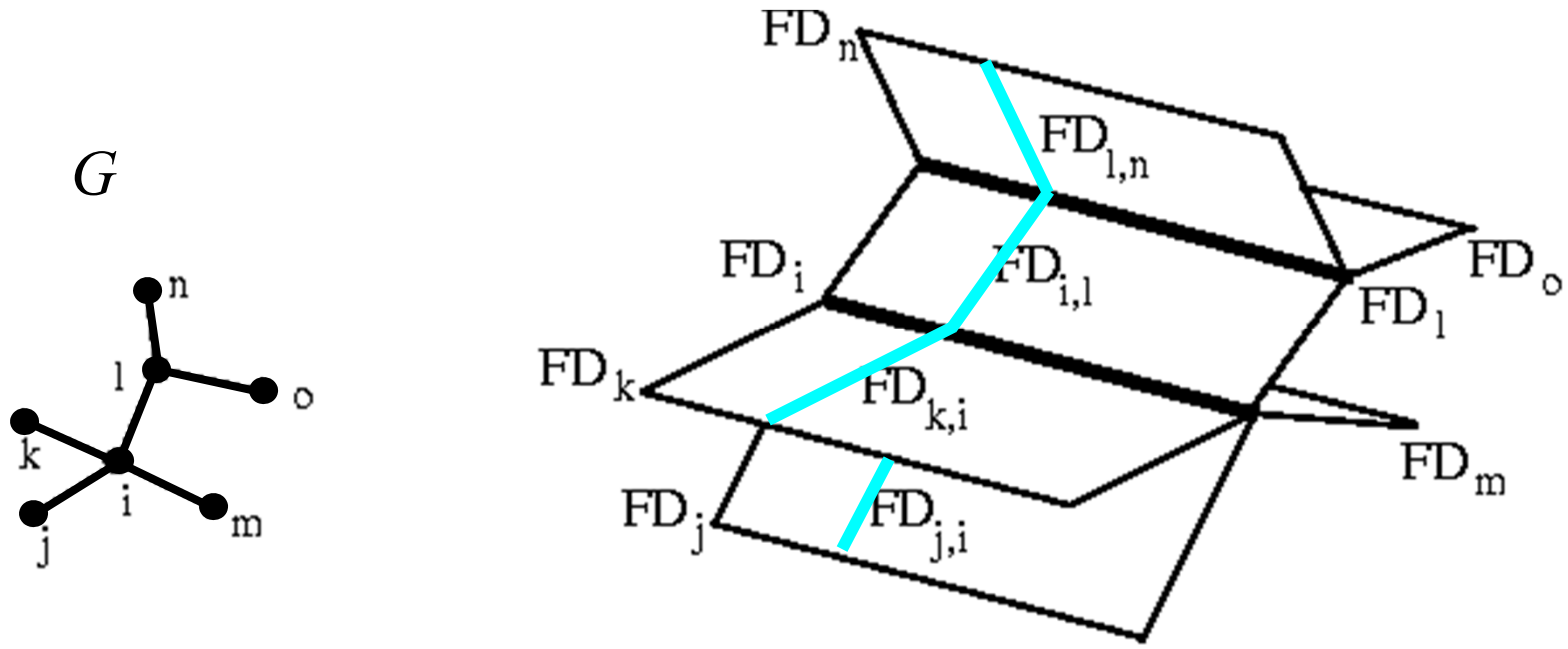

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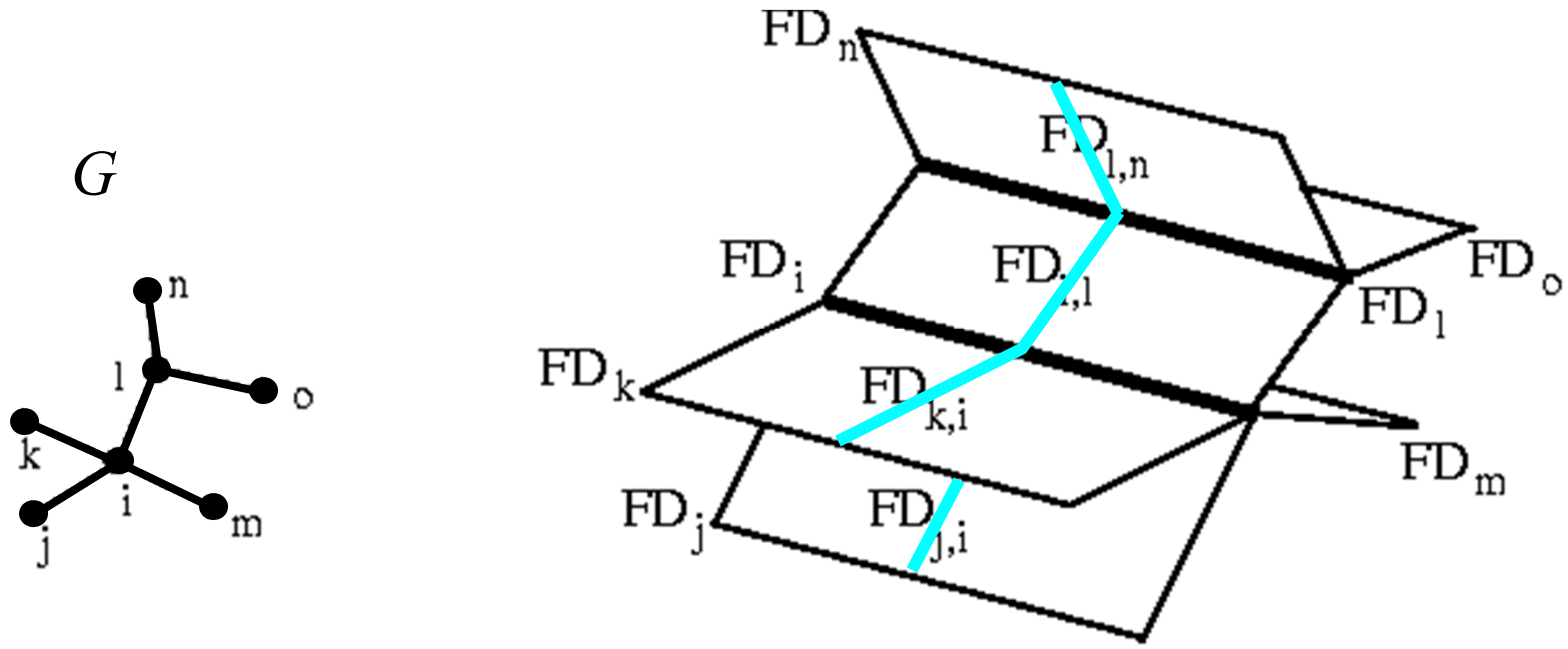

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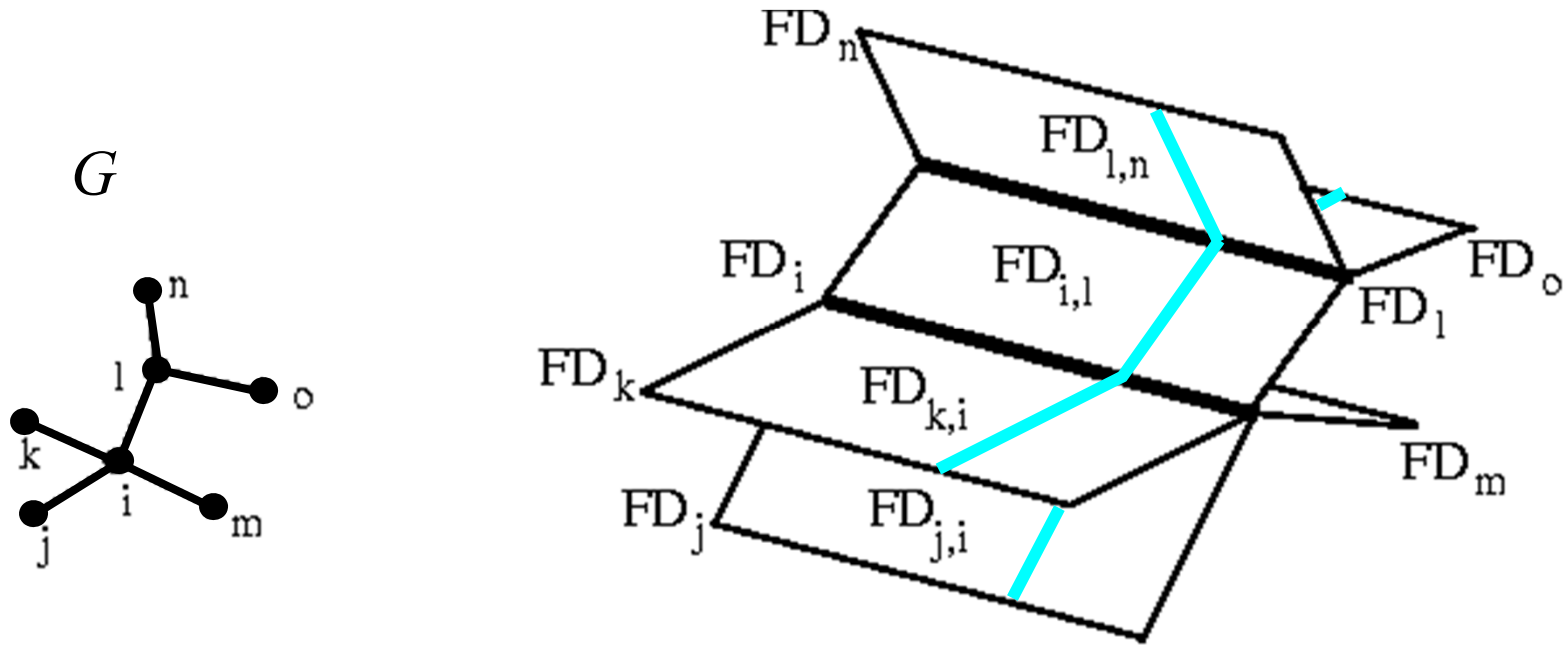

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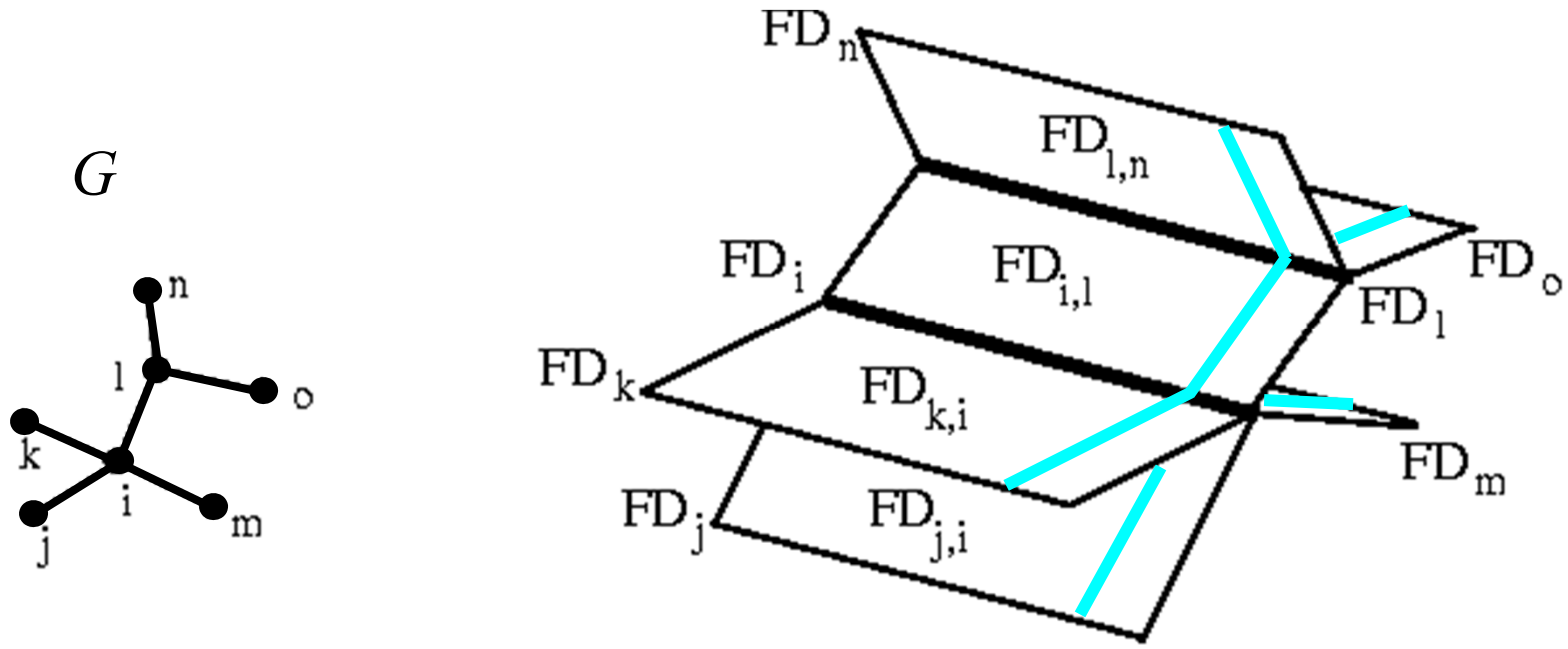

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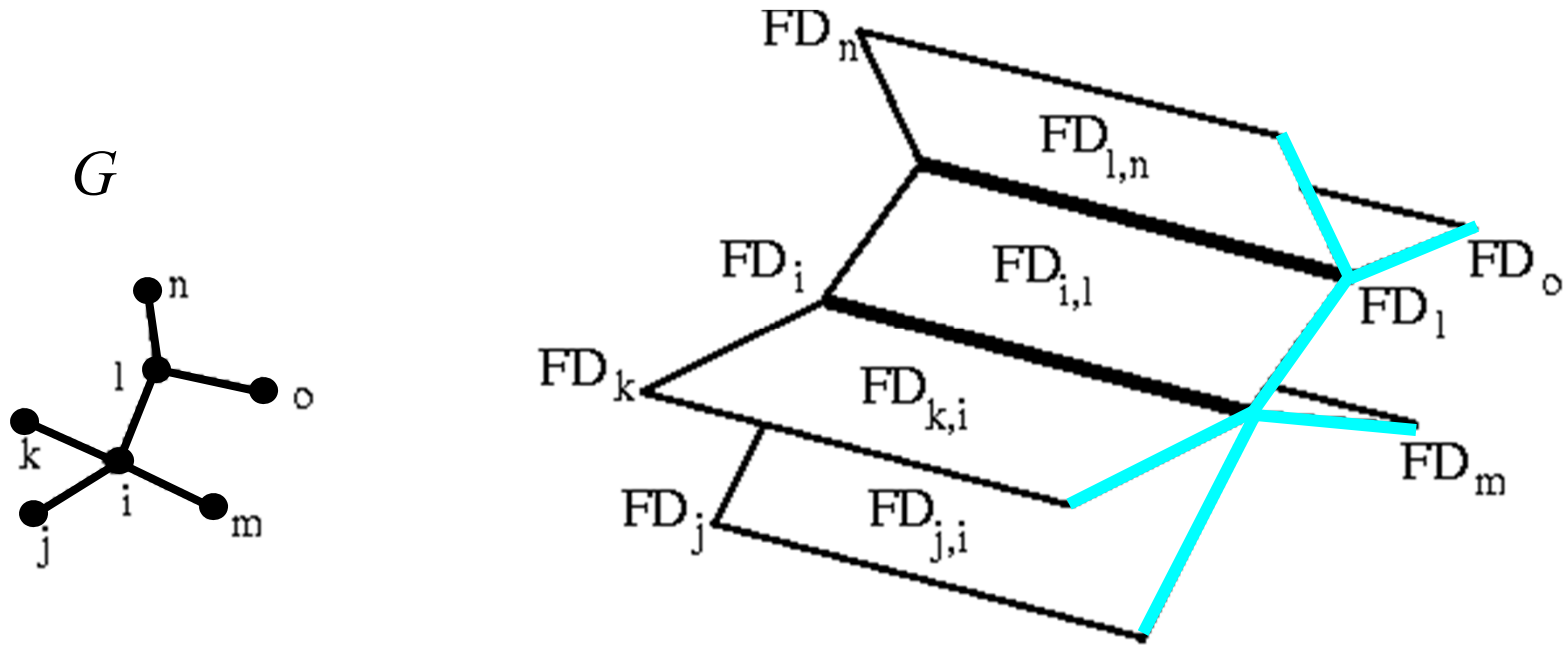

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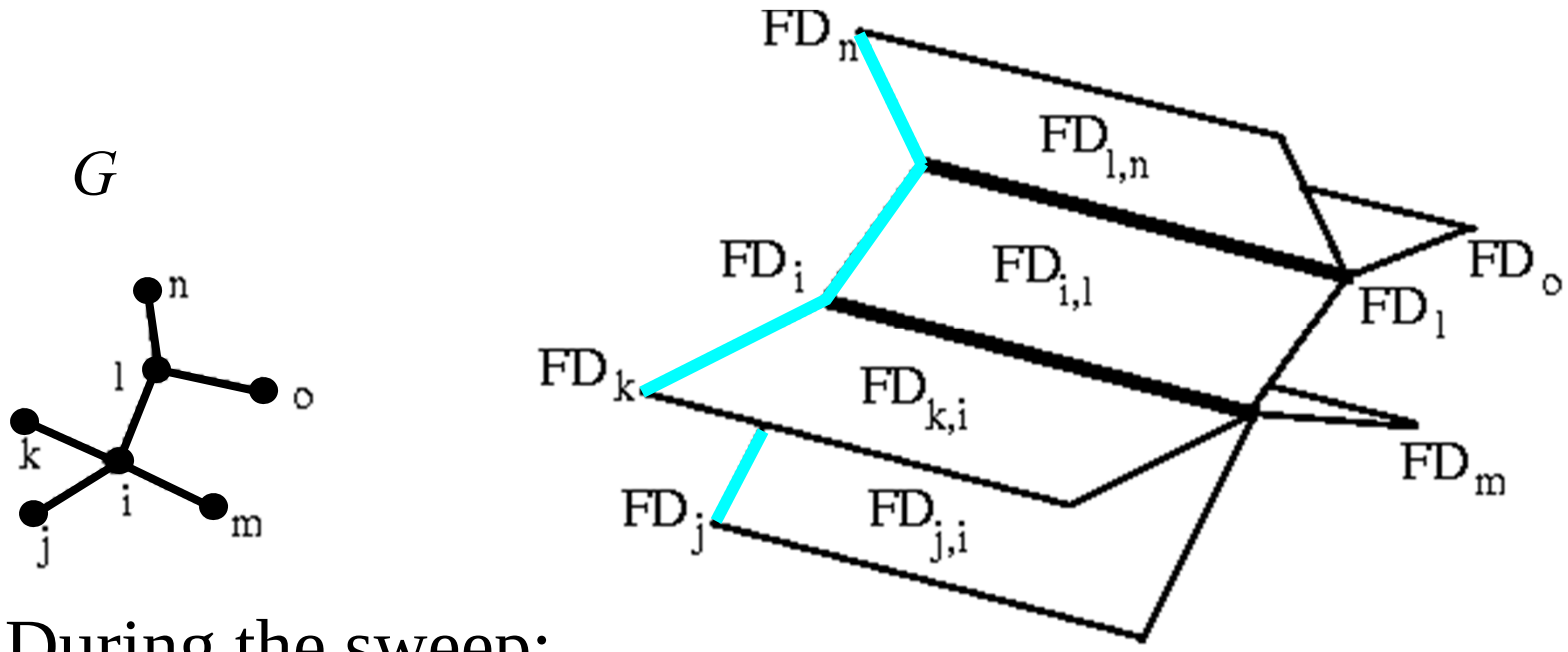
Compute Reachable Points



- During the sweep:

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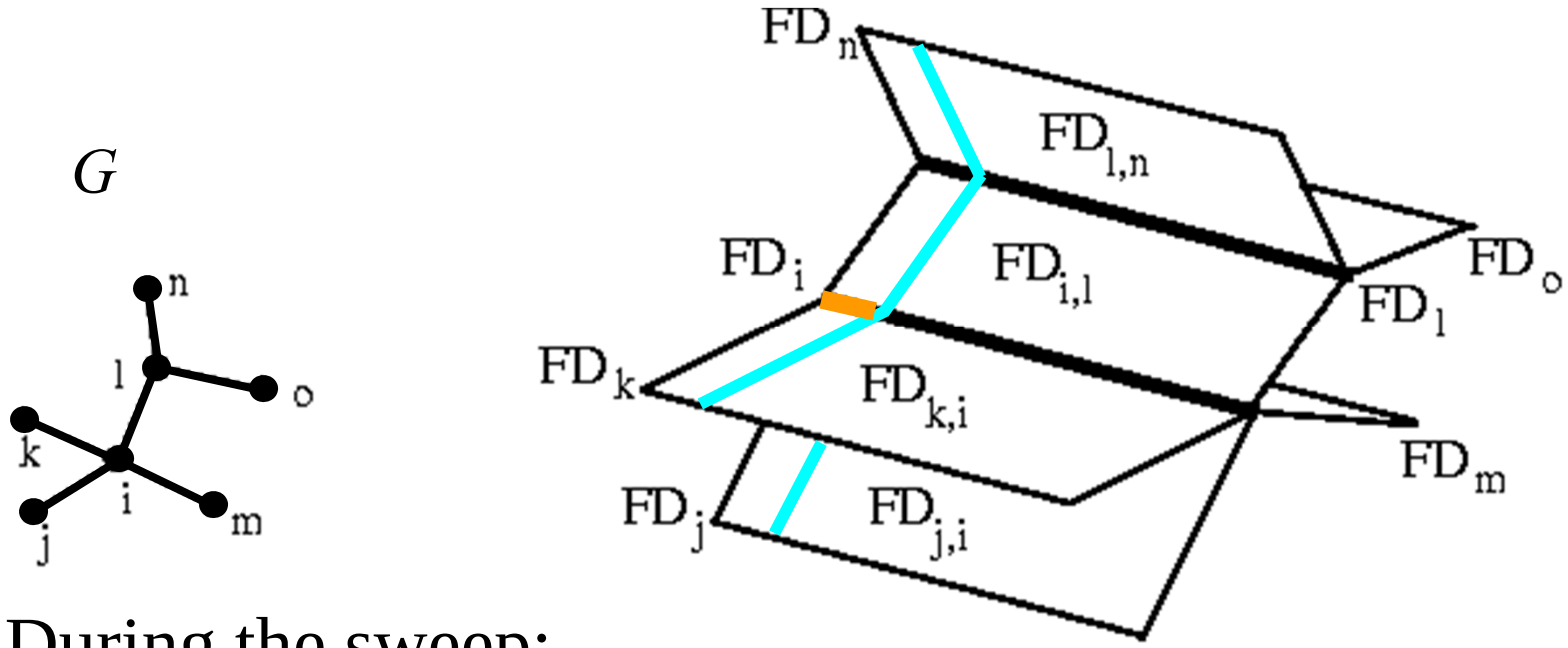

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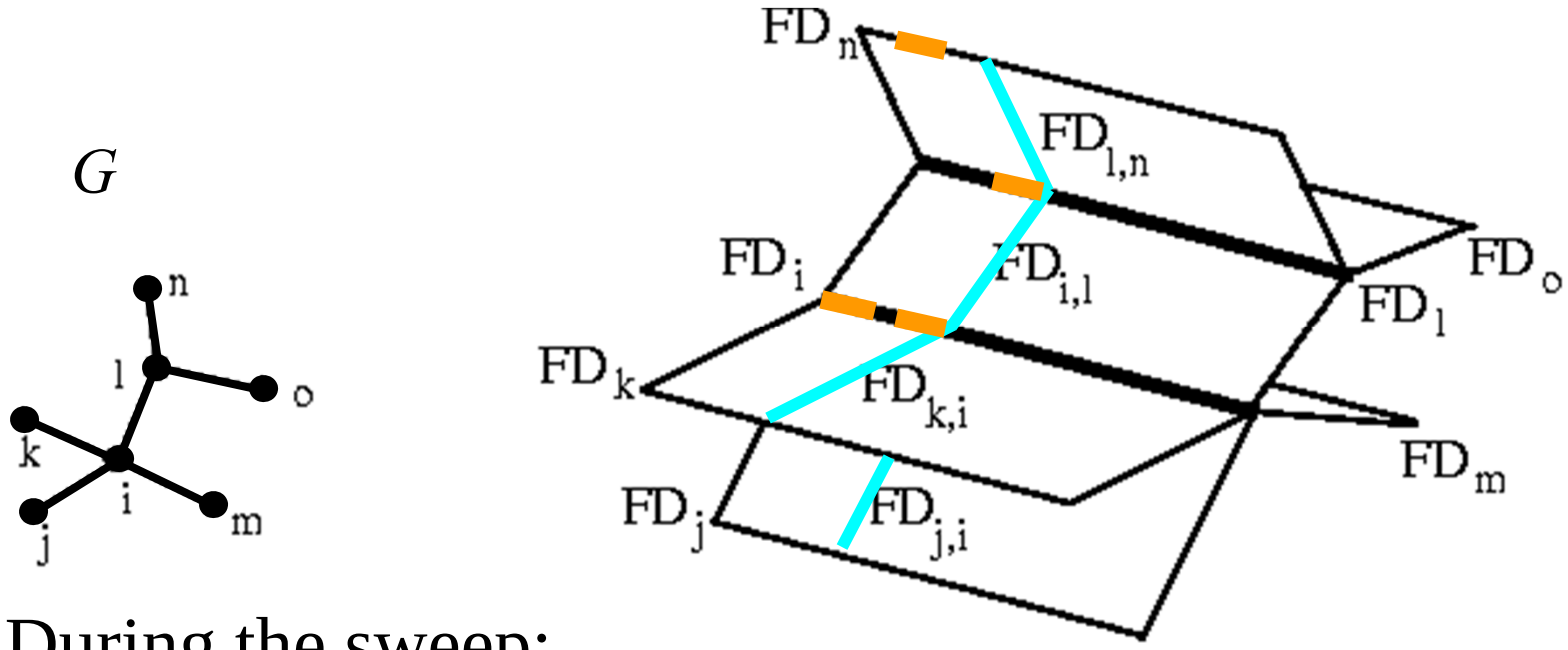

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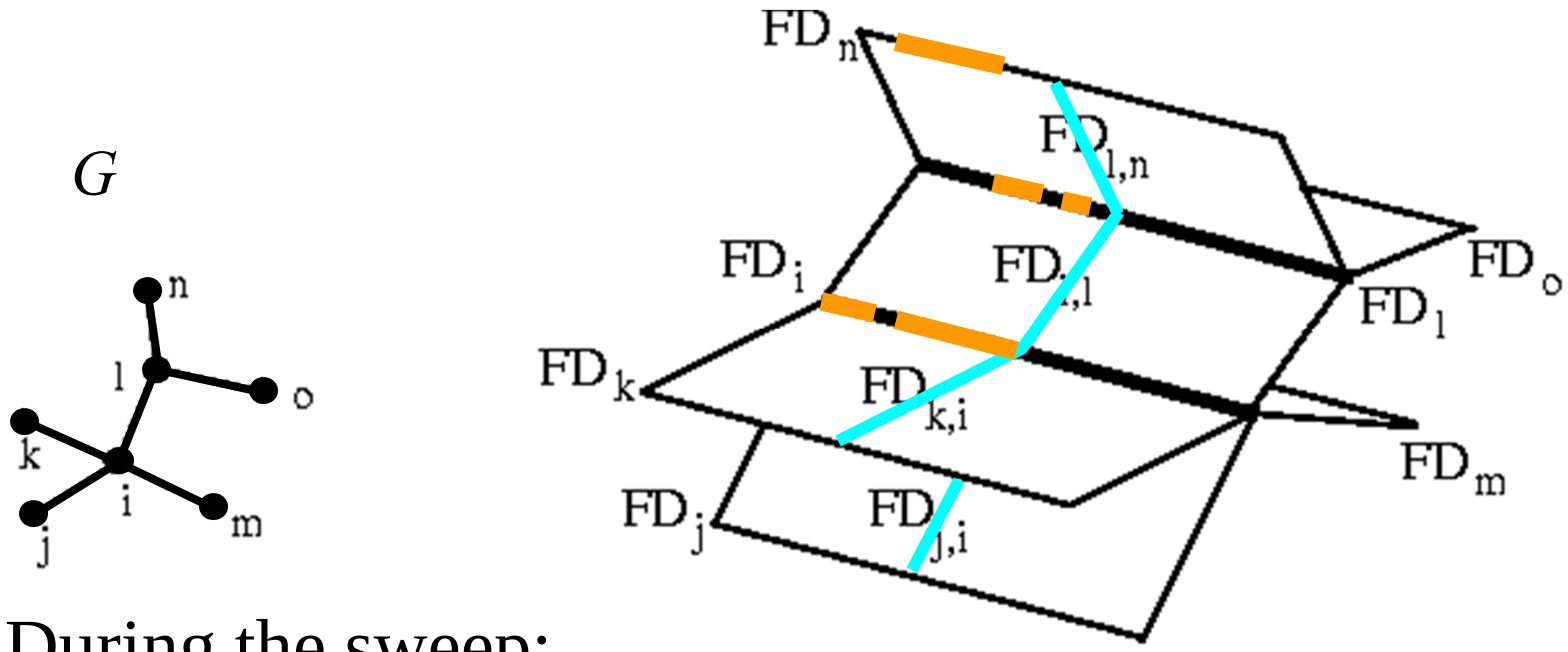

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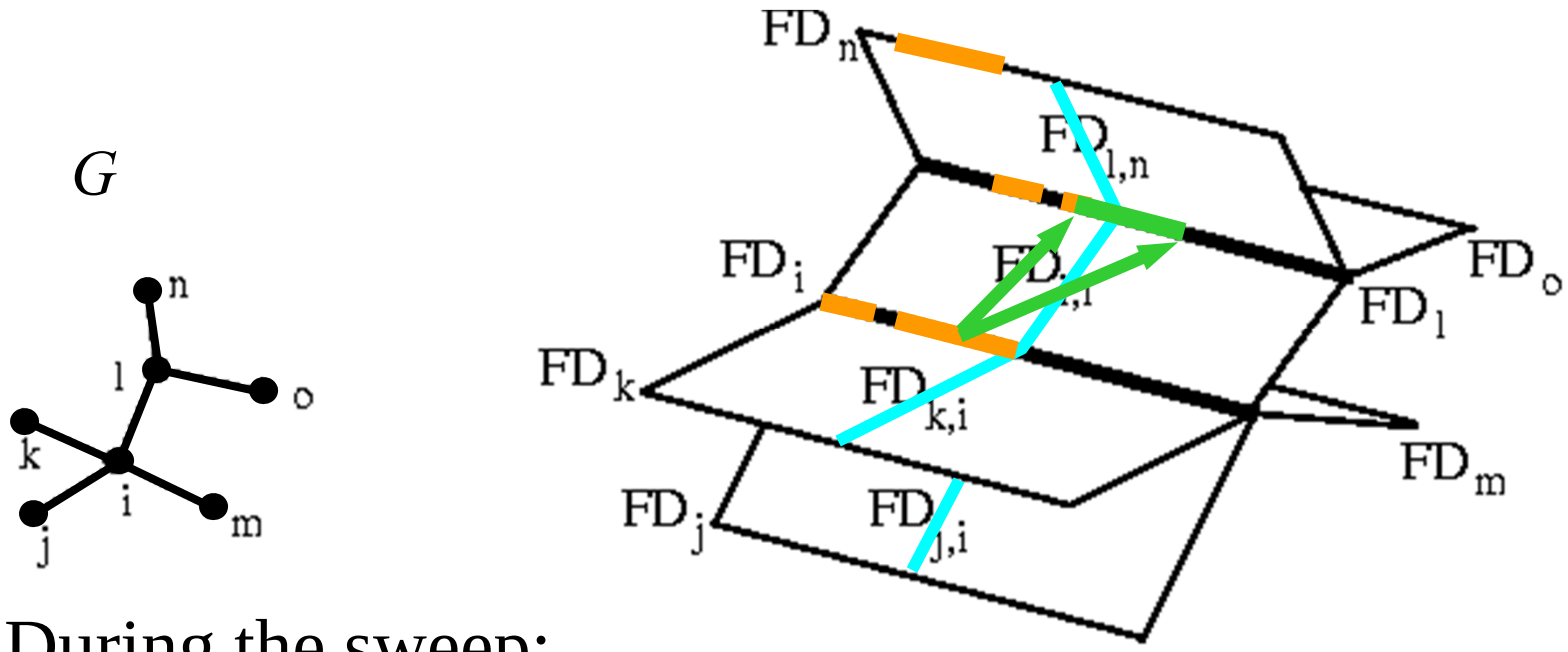

- During the sweep:
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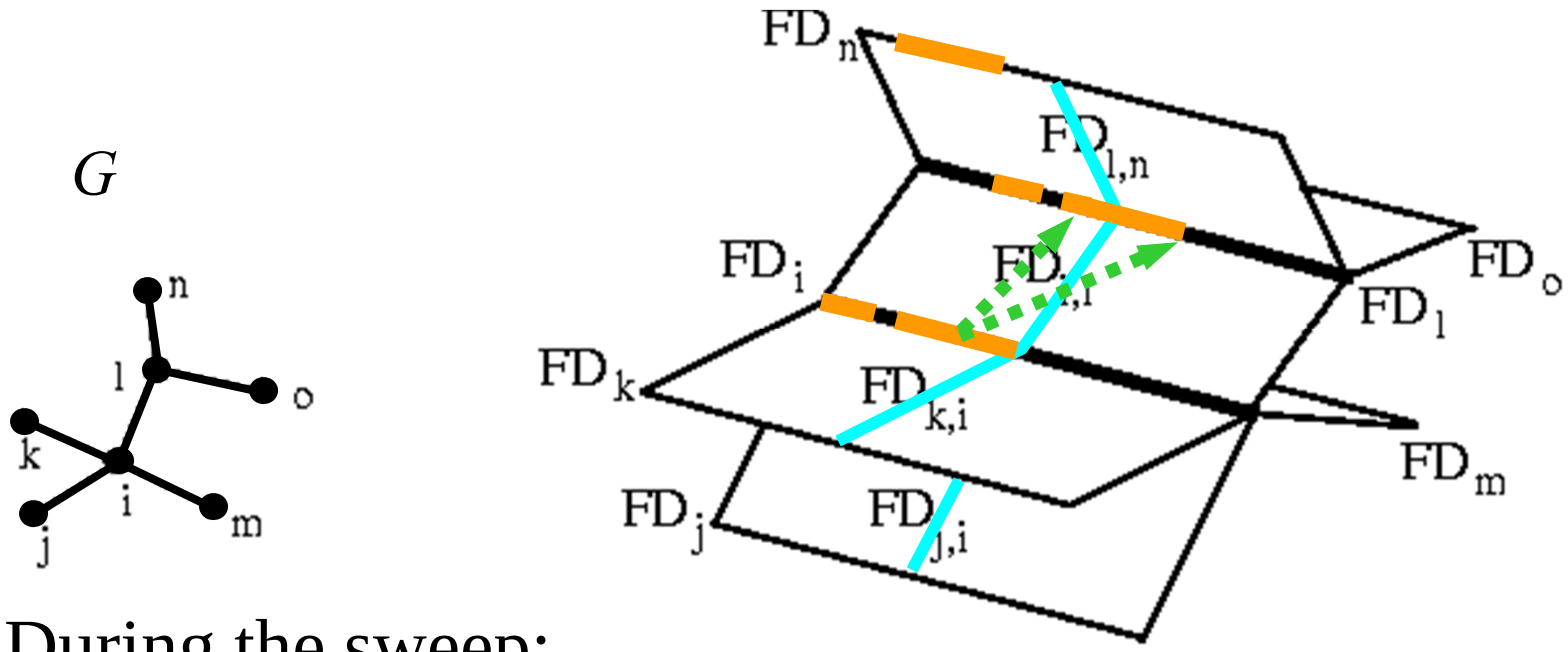

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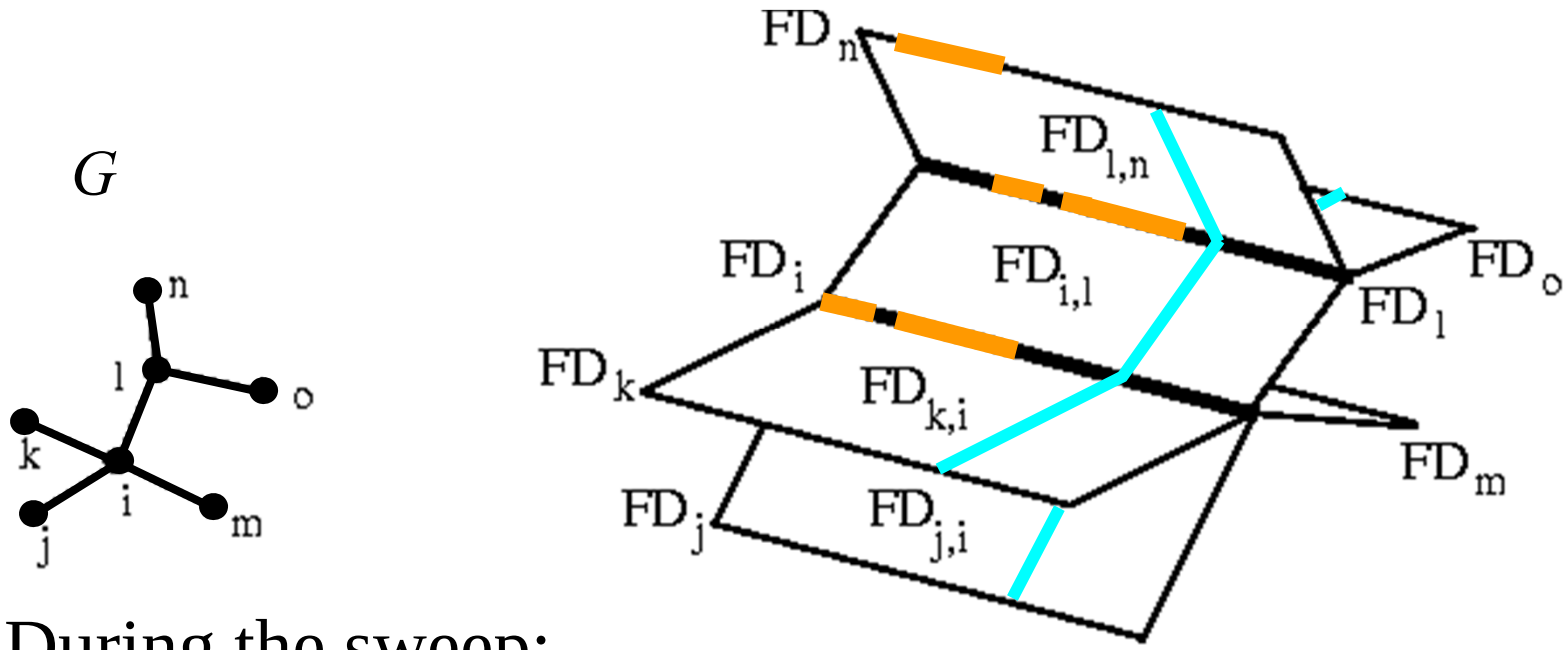

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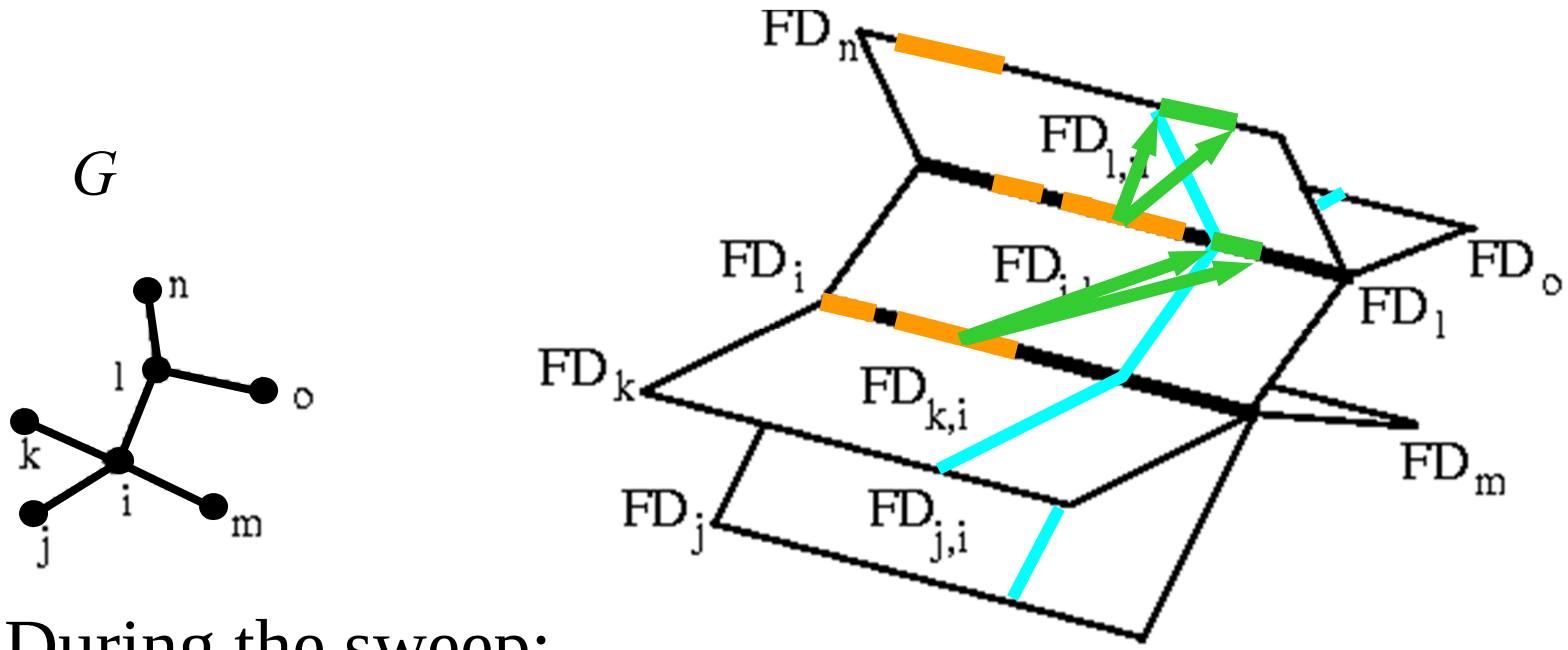

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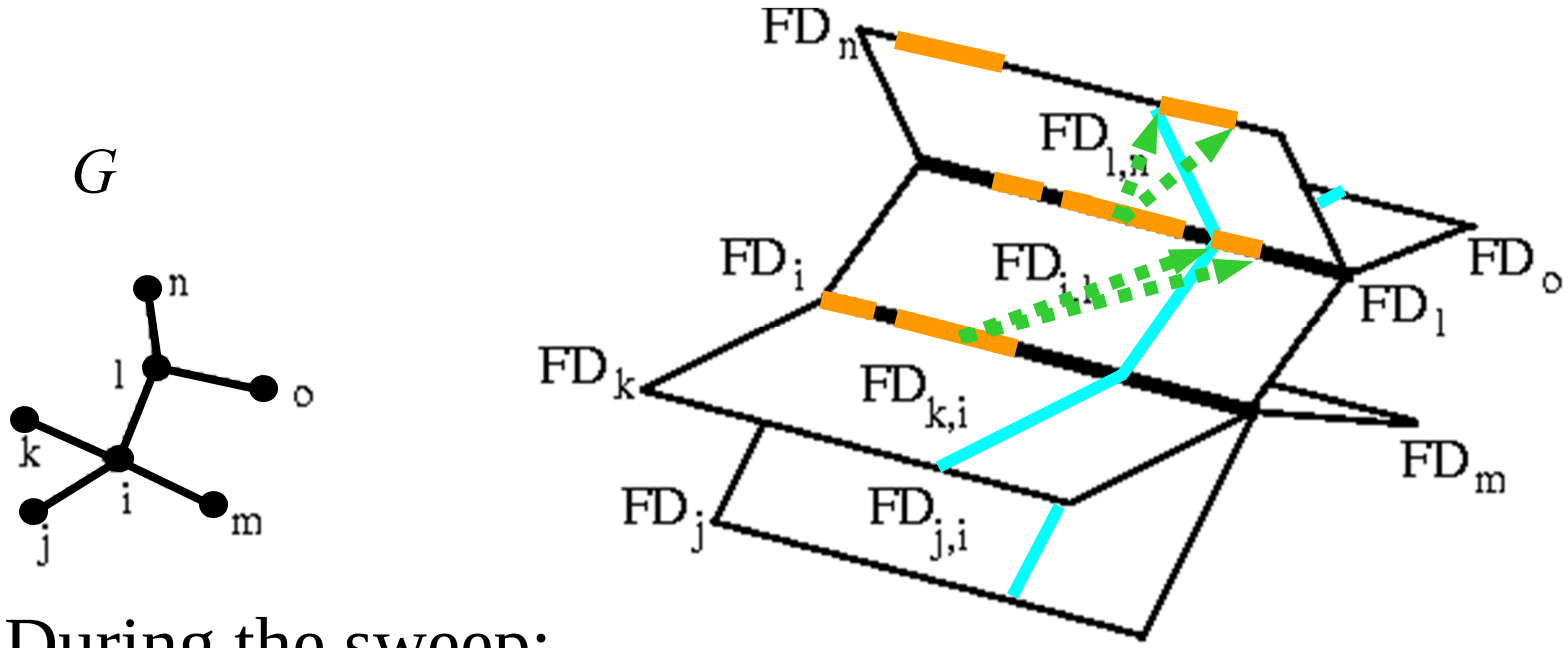

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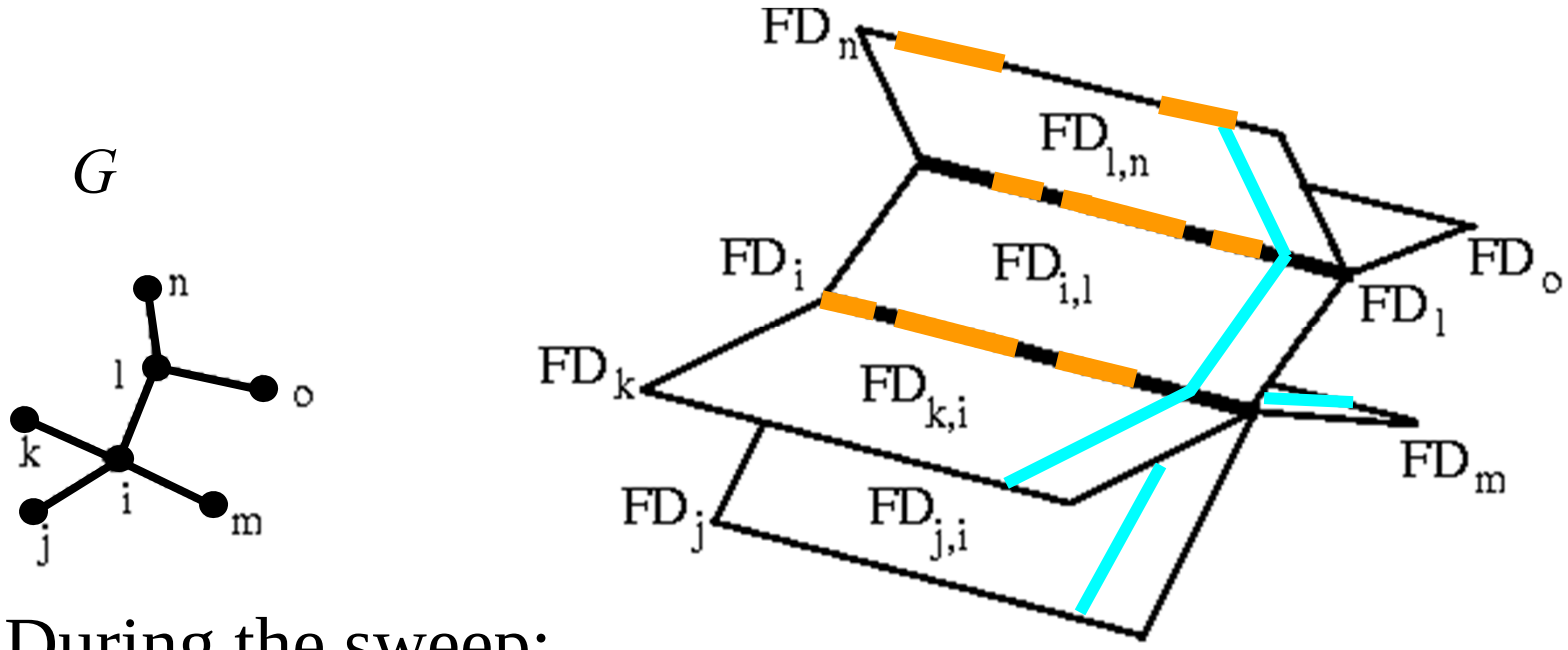

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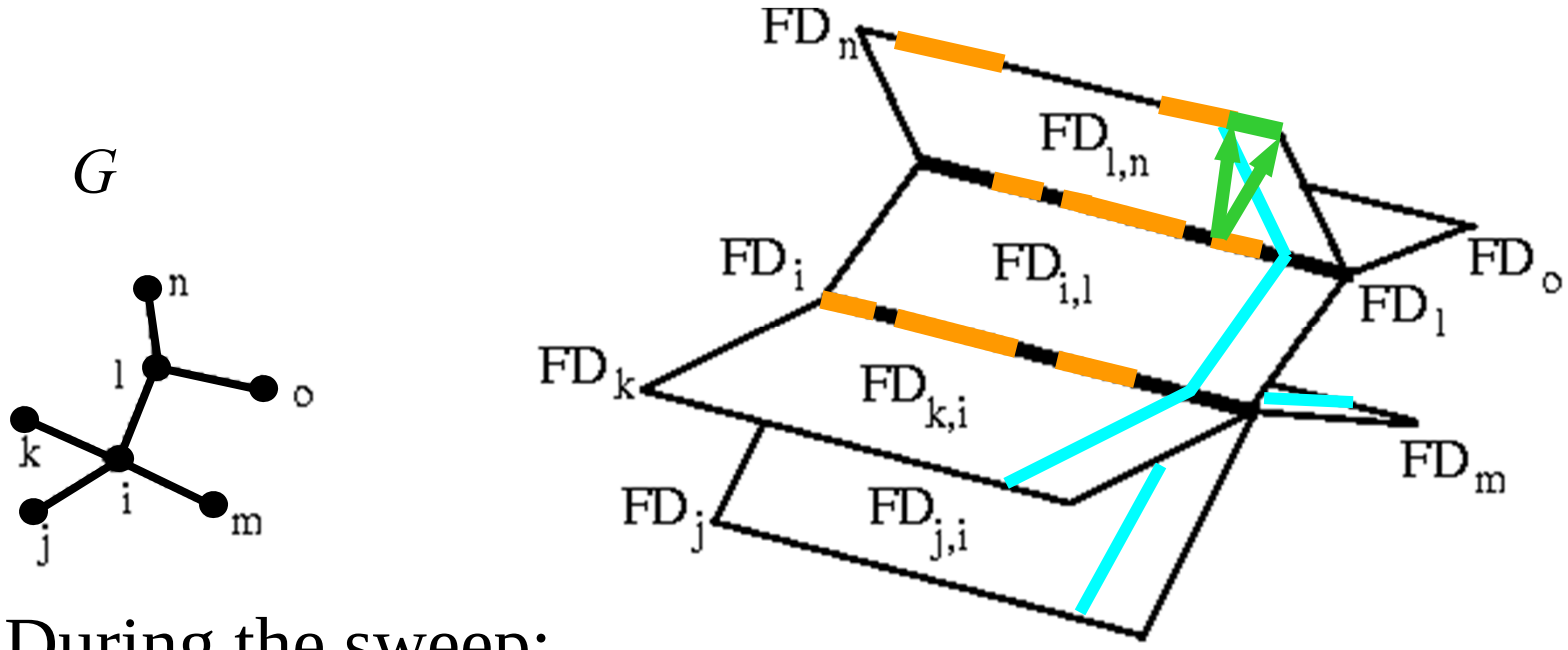

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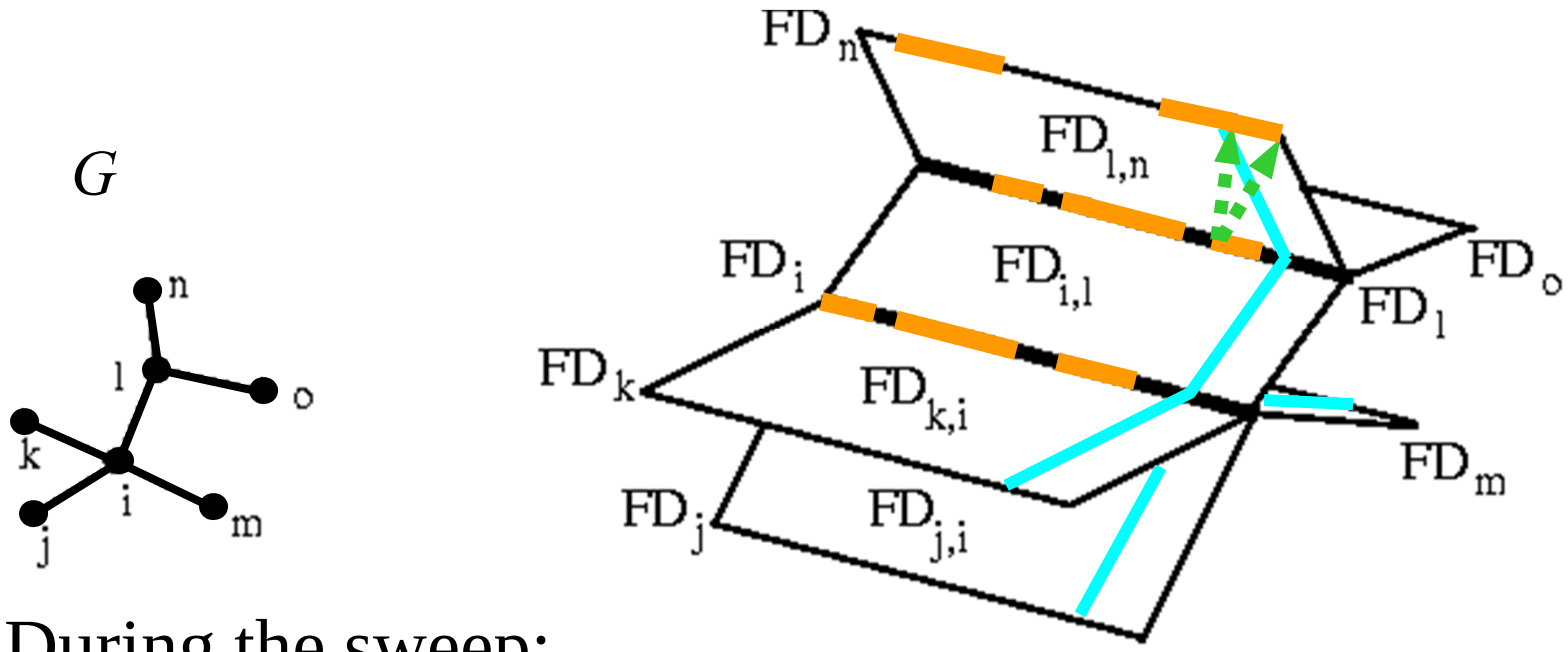

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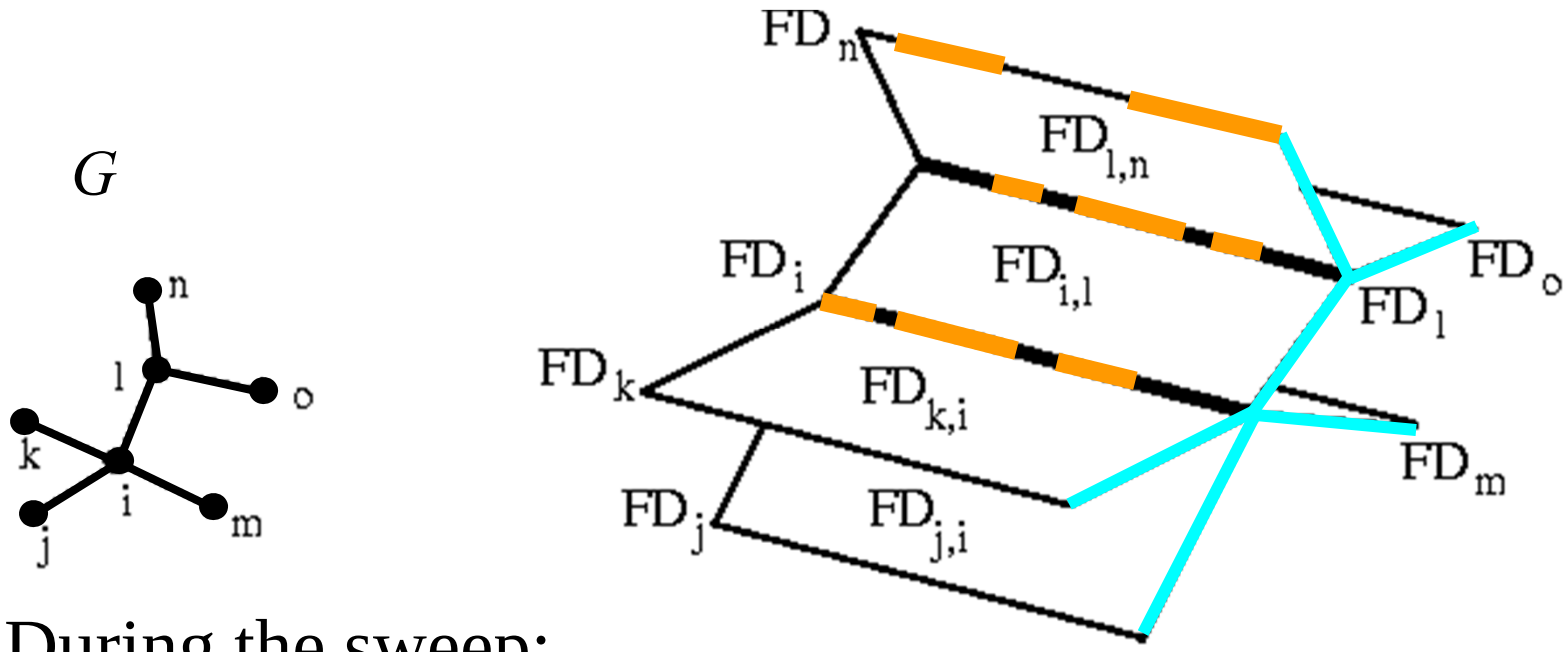

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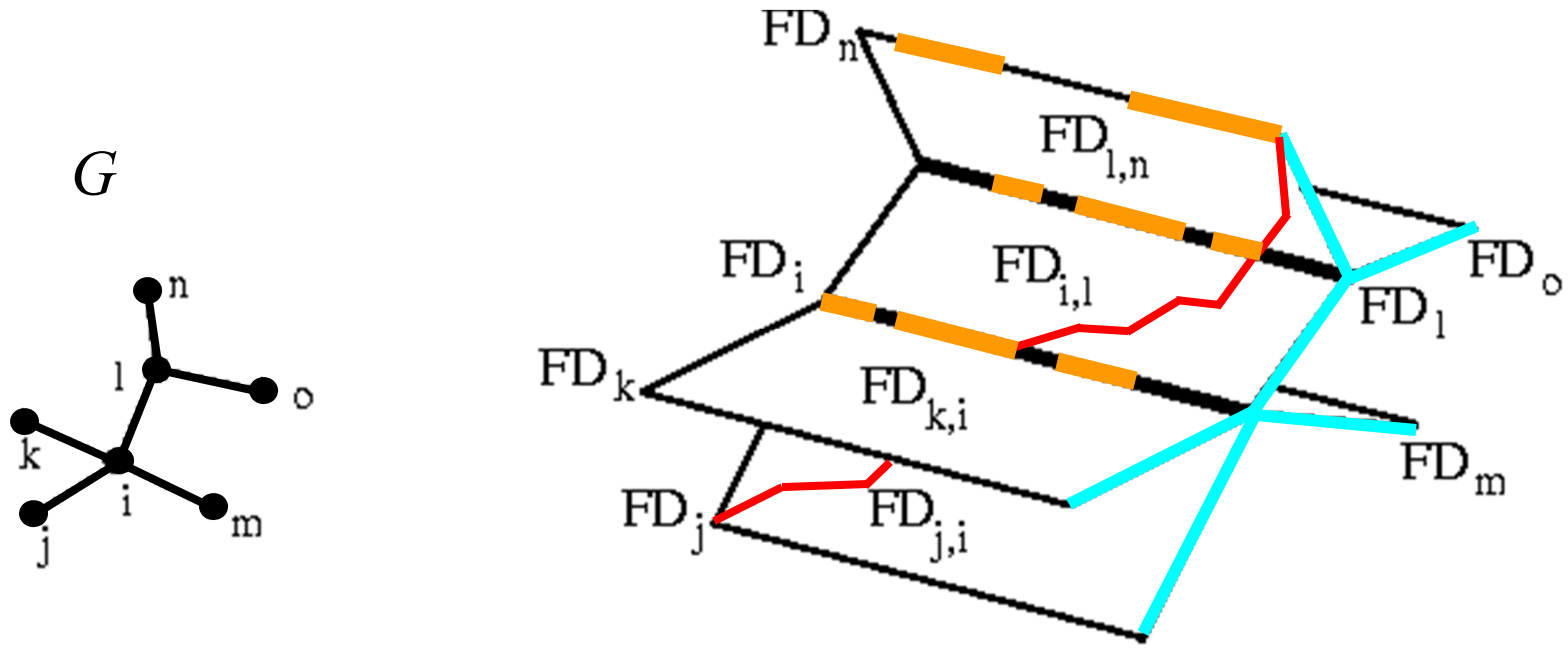
Backtracking



- After the sweep:
  - Construct a monotone path via backtracking

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Map-Matching

- Algorithm for decision problem takes O(mn log(mn)) time and O(mn) space.
- Optimization problem with parametric search: O(mn $\log^2$(mn)) time

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