CMPS 3130/6130 Computational Geometry – Spring 15

4/9/15

6. Homework

Due **4/23/15** before class

1. **Upper halfplanes (4 points)**
   Can the intersection of a finite number of upper halfplanes be empty? Justify your answer.

2. **Railway Tracks (8 points)**
   On n parallel railway tracks n trains are going with constant speeds $v_1, \ldots, v_n$. At time $t = 0$ the trains are at positions $k_1, \ldots, k_n$.

   Give an $O(n \log n)$ time algorithm that detects all trains that at some moment in time are leading.
   (Hint: Use halfplane intersection.)

3. **Linear Separator (8 points)**
   Let $R = \{r_1, \ldots, r_m\}$ be set of m red points, and let $B = \{b_1, \ldots, b_n\}$ be a set of n blue points in the plane. A line l is called a **linear separator** if all points of R lie on one side of l and all points of B lie on the other side. (You may assume appropriate general position, and may disregard points that lie exactly on the line.)

   Use point-line duality to develop an algorithm for this problem which runs in expected linear time. *(Hint: Linear Programming.)*

4. **Range Tree Construction (10 points)**
   (a) (3 points) Describe a recursive algorithm that constructs a 1D range tree for a **sorted** set of n numbers in $O(n)$ time.
   (b) (1 point) Same as above, but for a set of n unsorted numbers. Your algorithm should run in $O(n \log n)$ time.
   (c) (6 points) Describe a recursive algorithm that constructs a 2D range tree for a set of n two-dimensional points in $O(n \log n)$ time.
   (Hint: Use a bottom-up approach, and the merge-routine from mergesort.)

5. **Smallest Rectangle Queries (10 points)**
   Let P be a set of n points in the plane; you may assume that they are in general position. Devise a data structure of size $O(n \log n)$ to answer queries of the following form in $O(\log^2 n)$ time:

   > Given a vertical line segment s and an integer k, find the smallest rectangle that has s as its left side and which contains at least k points. If no such rectangle exists then indicate this.

   (Hint: Use 2D range trees with fractional cascading. Where does the extra log-factor come from in the query time?)

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6. **Convex Hull of Intersections (graduate; 10 points)**
Let $\mathcal{L}$ be a set of n lines in the plane, no two of which are parallel. Let S be the set of all $O(n^2)$ intersection points between any two lines in $\mathcal{L}$.

   (a) (5 points) Give an $O(n \log n)$ time algorithm to compute an axis-parallel rectangle that contains S.

   (b) (5 points) Give an $O(n \log n)$ time algorithm that computes $CH(S)$.

   (Hint: Your algorithms cannot compute all points in S explicitly. Sort all lines by slope, and prove that it is enough to consider only a certain subset of intersection points.)