CMPS 3130/6130 Computational Geometry – Spring 15

3/19/15

5. Homework

Due **4/9/15** before class

1. **Clique Complex** ($8 = 5 + 3$ **points)**
   A clique complex K corresponding to a graph G is the maximal abstract simplicial complex using G as the one-skeleton. That is, the vertices and edges of G are the vertices and edges of K. Higher dimensional simplices are created as follows: for each clique C in G, add a k-simplex to K consisting of the vertices in C.

   (a) Prove that a clique complex is an (abstract) simplicial complex.
   (b) Give an example of a simplicial complex that is not a clique complex.

2. **Connectivity** (10 **points)**
   Let $V = \{v_0, \ldots, v_k\} \subset \mathbb{R}^2$. Give an algorithm to compute the minimum radius r for which $\mathbb{B} = \bigcup_{v \in V} B_r(v)$ is connected. Be sure to give the complexity of this algorithm.

3. **Minimum Enclosing Ball** (4 **points)**
   Let P be a set of points in $\mathbb{R}^2$ and $r \in \mathbb{R}$. Prove that $\cap_{p \in P} B_r(p)$ is non-empty if and only if there exists $s \in \mathbb{R}^2$ such that $P \subset B_r(s)$. [Note: An implication of this result is that we can use an algorithm to compute the radius of the smallest enclosing disk of a set of points to determine whether or not a simplex is in the Čech complex.]

4. **Graph Genus** ($10 = 5 + 5$ **points)**
   Recall that an orientable (compact) two-manifold is either a sphere $\mathbb{S}^2$, the torus $\mathbb{T}^2$, or the connected sum of g tori:

$$\mathbb{T}^2 \underbrace{\#\mathbb{T}^2\# \cdots \#\mathbb{T}^2}_{g-1 \text{ times}}.$$

   The number g is known as the genus of the orientable manifold (where, if $g = 0$, we have the sphere; see also the paragraph on handle-body decomposition in the notes). The genus of a graph is the smallest g such that the graph can be embedded on a compact orientable manifold of genus g.

   (a) What is the genus of the complete graph on five vertices, K_5? Show or explain.
   (b) What is the genus of $K_{3,3}$ (the complete bipartite graph between two sets of three vertices)? Show or explain.

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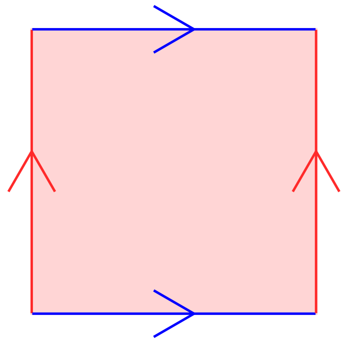
5. **Computing Invariants** ($8 = 2 + 2 + 2 + 2$ **points)**
   Consider the following fundamental polygon:



   (a) What is this (topological) space? [Hint: we have seen it in the lectures.]
   (b) Provide a valid simplicial complex that represents the same underlying space.
   (c) What is the Euler characteristic of it? (Be sure to show how you got your answer).
   (d) Is it orientable? Explain your anwer.

6. **Different Norms (graduate;** $10 = 5 + 5$ **points)**
   Let $V = \{v_0, \dots, v_n\}$ be a collection of $n+1$ points in $\mathbb{R}^d$. In lecture, we used Euclidean-balls to define Čech and Alpha complexes. Consider instead, the ℓ_∞ balls. That is, we define the distance between a vertex v_i and a point $x \in \mathbb{R}^d$ as follows:

$$d(v_i, x) = ||v_i - x||_\infty = \max_{j=1\dots d} |v_i(j) - x(j)|,$$

   where $v_i(j)$ and $x(j)$ are the j-th coordinates of v_i and x, respectively. Prove that the Čech complex using this metric is a clique complex for some graph. Show (by counter-example) that this is not necessarily true when using Euclidean balls.

Note: If you would like to discuss any of these homework problems with Brittany Fasy, please email bfasy@tulane.edu to set up a time.