CMPS 3130/6130 Computational Geometry – Spring 15

3/5/15

4. Homework

Due **3/19/15** before class

1. **Worst-Case DT Runtime (8 points)**
   Give an example that shows that the worst-case runtime of the randomized algorithm to compute the Delaunay triangulation of a set of n points in the plane is $\Omega(n^2)$. *(Hint: It might help to play with one of the Delaunay triangulation programs.)*

2. **Sum of Edge Lengths (8 points)**
   It appears that illegal edges are often long edges, so it is a natural question to ask whether the Delaunay triangulation might minimize edge lengths. Give an example which shows that the Delaunay triangulation of a point set is not always the triangulation with the minimum sum of edge lengths.

3. **Gabriel Graph (12 points)**
   Let P be a set of n points in the plane. The *Gabriel graph* $GG(P)$ is defined as follows: Two points $p, q \in P$ are connected by an edge in $GG(P)$ iff the circle with diameter pq does not contain any other point of P in its interior.

   (a) Prove that $DT(P)$ contains $GG(P)$. I.e., every edge in $GG(P)$ is also a Delaunay edge.

   (b) Prove that p and q are adjacent in $GG(P)$ if and only if the Delaunay edge between p and q intersects its dual Voronoi edge.

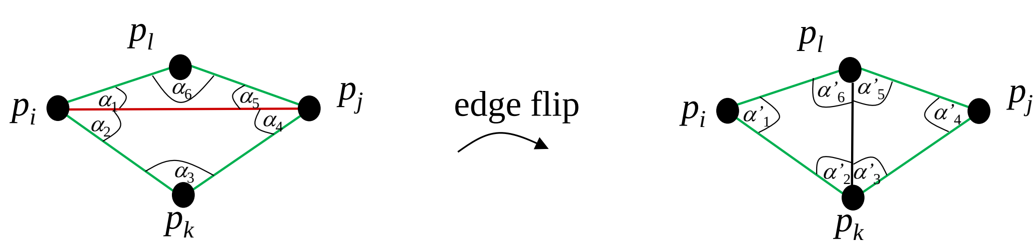
   (c) Give an $O(n \log n)$ time algorithm to compute $GG(P)$.

4. **Edge Flips (12 points)**

   (a) Show that any two triangulations of a convex polygon can be transformed into each other by a sequence of edge flips.

   (b) Show that any two triangulations of a planar point set can be transformed into each other by a sequence of edge flips.

5. **Angle Optimality (graduate; 10 points)**
   Prove the Lemma on slide 7 of the Delaunay I slides: Let $p_i, p_j, p_k, p_l \in P$. Then $\overline{p_i p_j}$ is illegal $\Leftrightarrow \min_{1 \leq i' \leq 6} \alpha_{i'} < \min_{1 \leq i' \leq 6} \alpha'_{i'}$.



Hint: Use Thales' theorem.