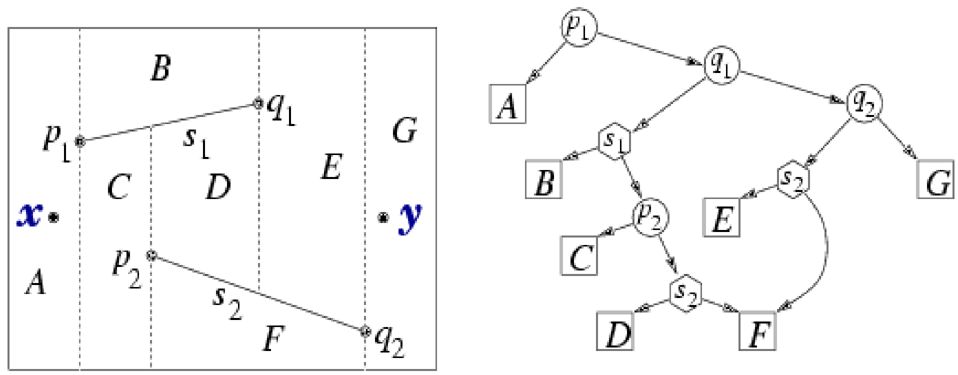
# CMPS 3130/6130 Computational Geometry – Spring 15

2/19/15

## 3. Homework

Due **3/5/15** before class

1. **Trapezoidal Map (8 points)**
   Consider the following instance of the trapezoidal map point location data structure. The left side shows the map, and the right side shows the corresponding DAG. Describe how the DAG is modified if the next segment to be added is $\overline{xy}$.




2. **Horizontal Ray Shooting (6 points)**
   Let $S$ be a set of $n$ disjoint line segments in the plane. We wish to answer the following kind of "horizontal ray shooting" queries efficiently: Given a query point $q$, determine the first segment hit by a bullet that starts in $q$ and is shot to the right along a horizontal ray. Describe a data structure that takes $O(n)$ space and can be constructed in $O(n \log n)$ preprocessing time, such that ray shooting queries can be answered in $O(\log n)$ time. All complexities may be in expectation. *(Hint: Use point location.)*

3. **Parabolic Arc (6 points)**
   Give an example where the parabola defined by some site $p_i$ contributes more than one arc to the beach line. Can you give an example where it contributes a linear number of arcs?

4. **Hausdorff Distance (10 points)**
   Let $A$ and $B$ be two point sets in the plane with $m = |A|$ and $n = |B|$. The *directed Hausdorff distance* $h(A, B)$ is defined as $h(A, B) = \max_{a \in A} \min_{b \in B} d(a, b)$, where $d(.,.)$ is the Euclidean distance. The *(undirected) Hausdorff distance* $H(A, B)$ is defined as $H(A, B) = \max\{h(A, B), h(B, A)\}$. Show that the undirected Hausdorff distance can be computed in $O((n + m)\log(n + m))$ time.

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5. **Weighted Voronoi Diagrams (10 points)**
Let $P = \{p_1, \ldots, p_n\} \subseteq \mathbb{R}^2$, and let $w_i > 0$ be the *weight* of point site $p_i$, for each $i = 1, \ldots, n$. In the *additively weighted* Voronoi diagram the Voronoi cell for $p_i$ is defined as

$$V_{add}(p_i) = \{q \in \mathbb{R}^2 \mid w_i + ||p_i - q|| < w_j + ||p_j - q|| \text{ for all } p_j \in P \setminus \{p_i\}\}$$

In the *multiplicatively weighted* Voronoi diagram the Voronoi cell for $p_i$ is defined as

$$V_{mult}(p_i) = \{q \in \mathbb{R}^2 \mid w_i * ||p_i - q|| < w_j * ||p_j - q|| \text{ for all } p_j \in P \setminus \{p_i\}\}$$

Show how the bisectors look like for both kinds of weighted Voronoi diagrams, and give some examples of Voronoi diagrams for each case. You are welcome to research on the web as long as you give references.

6. **Reverse Voronoi (graduate; 10 points)**
Suppose we are given a subdivision of the plane into $n$ convex regions. We suspect that this subdivision is a Voronoi diagram, but we do not know the sites. Develop an algorithm that finds a set of $n$ point sites whose Voronoi diagram is exactly the given subdivision, if such a set exists.