CMPS 3130/6130 Computational Geometry – Spring 15

1/22/15

1. Homework

Due **2/5/15** before class

1. **Point location data structure for convex polygons (10 points)**
   Describe an *efficient* data structure to preprocess a convex n-gon P in the plane, such that given a query point q it can be answered in $O(\log n)$ time whether $q \in P$ or not. Analyze the preprocessing time and the space requirement of your data structure, and make both as efficient as possible.

2. **Line segment intersection (10 points)**
   Given two line segments $\overline{ab}$ and $\overline{cd}$ in the plane, where $a, b, c, d \in \mathbb{R}^2$. The goal is to test them for intersection.

   (a) (5 points) Express each line segment as a convex combination, and use this representation to determine if the two line segments intersect. *(Hint: Each line segment is now a function. Equate them, and solve a linear equation system.)*

   (b) (5 points) Explain how you can use one or more orientation tests to test if the two line segments intersect. *(Hint: Case analysis. Draw pictures of examples, and determine important configurations of a, b, c, d.)*

3. **Lower bounds (9 points)**
   Consider the following problems:

   SORTING: Given a set $X = \{x_1, \ldots, x_n\}$ of n numbers, output the same numbers in non-decreasing order.

   ELEMENT UNIQUENESS: Given a set $X = \{x_1, \ldots, x_n\}$ of n numbers, are there i, j, with $i \neq j$, such that $x_i = x_j$?

   CLOSEST PAIR: Given a point set $P = \{p_1, \ldots, p_n\} \in \mathbb{R}^2$, output the closest pair of points in P.

   ALL NEAREST NEIGHBORS: Given a point set $P = \{p_1, \ldots, p_n\} \in \mathbb{R}^2$. Compute for each point in P its *nearest neighbor* in P (i.e., point at minimum distance).

   (a) Prove a lower bound of $\Omega(n \log n)$ for SORTING, by reducing from ELEMENT UNIQUENESS (i.e., by using the knowledge that ELEMENT UNIQUENESS has a lower bound of $\Omega(n \log n)$).

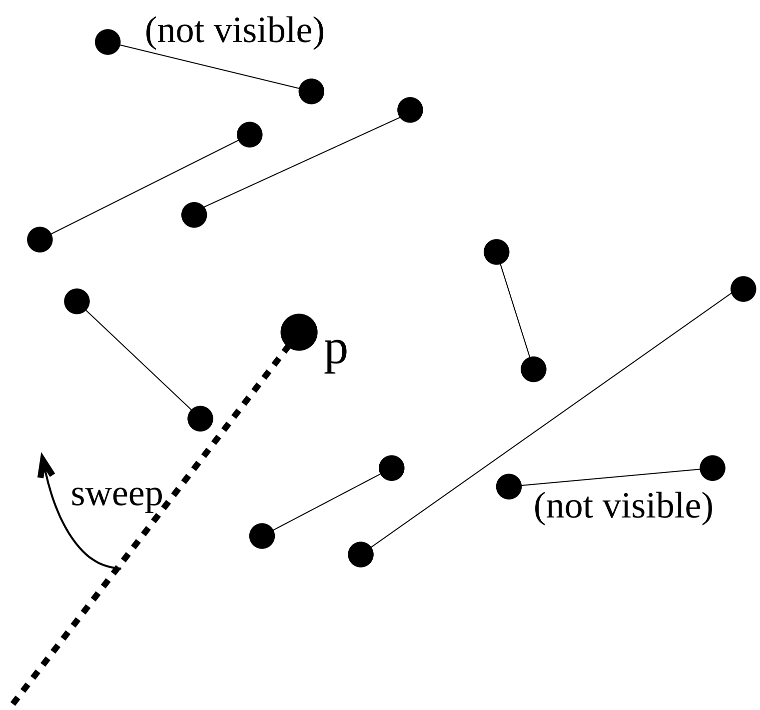
   (b) Prove a lower bound of $\Omega(n \log n)$ for CLOSEST PAIR by reducing from an appropriate problem.

   (c) Prove a lower bound of $\Omega(n \log n)$ for ALL NEAREST NEIGHBORS by reducing from an appropriate problem.

FLIP OVER TO BACK PAGE $\Longrightarrow$

4. **Visible segments sweep (11 points)**
Let S be a set of n disjoint line segments in the plane, and let p be a point not on any of the line segments of S. We wish to determine all line segments of S that P can see, i.e., all line segments of S that contain some point q so that the open segment $\overset{\vdash\dashv}{pq}$ does not intersect any line segment of S.

Give an $O(n \log n)$ time algorithm for this problem that uses a rotating half-line with its endpoint at p.



5. **Convex Hull (graduate; 10 points)**
Let $S \subseteq \mathbb{R}^2$ be a finite point set. Denote with $CH(S)$ the convex hull of S.

   (a) Let P be the convex polygon whose boundary vertices are points in S and that contains all points in S. Prove that P is uniquely defined and that $CH(S) = P$.

   (b) Let $C(S)$ be the set of all convex combinations of points in S. Prove that $C(S) = CH(S)$. *(Hint: Prove that $C(S)$ is convex.)*