CMPS 3120 Computational Geometry – Spring 13

4/16/13

6. Homework

Due **Tuesday 4/30/13** before class

1. **Nesting segment trees and range trees (10 points)**
   In class we used a *segment-range* tree to solve the 2-dimensional windowing problem. This two-level tree consists of a segment tree as the primary tree, and it stores in each node of the primary tree a link to a secondary tree which is implemented as a range tree.

   Now consider defining a *range-segment* tree which has a range tree as the primary tree and segment trees as the secondary trees. We can also define a *segment-segment* tree in a similar way, and *range-range* trees we have already studied in class.

   Compare all four data structures and argue what kinds of problems each can be used to solve. Analyze and compare the query times, construction times, and space complexities.

2. **All circular orders (10 points)**
   The $O(n^2 \log n)$-algorithm to construct the visibility graph performs circular line-segment visibility sweeps (homework 1, question 4) around every obstacle vertex $v \in V$. Each such sweep begins with computing a circular order of all vertices $V \setminus \{v\}$ around v, which in homework 1 we performed in $O(n \log n)$ time.

   (a) Show that you can use point-line duality to compute all circular orderings around all vertices in V in total $O(n^2)$ time (which is faster than performing n sorts of $O(n \log n)$ time each).

   (b) Why doesn't this speed-up suffice to compute the visibility graph in $O(n^2)$?

3. **Number of shortest paths (5 points)**
   Give an example n line-segment obstacles and two points a, b such that the number of different shortest paths between a and b is exponential in n. Also give the exponential number that you come up with.

   (Hint: It might help to assume that the line segments are open, i.e., endpoints are no obstacles. This means that two line segments can touch in a point, and a path is allowed to pass through this point.)

4. **Minkowski sums (10 points)**

   (a) Let P and Q be subsets of R^2 of m and n points, respectively. Show that $CH(P) \oplus CH(Q) = CH(P \oplus Q)$.

   (b) **[For extra credit only]** Same as in (a), just for R^3, or possibly for R^d.

   (c) Let A and B be convex subsets of R^3 with m and n vertices. Give an algorithm to compute $A \oplus B$. *(Hint: Use part (b).)*

   (d) **[For extra credit only]** Consider the 3-case theorem for computing the Minkowski sums of polygons (convex/convex, convex/non-convex, non-convex/non-convex) that we covered in class. Extend it to polyhedra in R^3.