CMPS 3120 Computational Geometry – Spring 13

4/3/13

5. Homework

Due **Tuesday 4/16/13** before class

1. **Railway Tracks (10 points)**
   On n parallel railway tracks n trains are going with constant speeds $v_1, \ldots, v_n$. At time $t = 0$ the trains are at positions $k_1, \ldots, k_n$.

   Give an $O(n \log n)$ time algorithm that detects all trains that at some moment in time are leading.
   (Hint: Use halfplane intersection.)

2. **Range Counting (5 points)**
   Show how to augment a 1D range tree of n elements such that range **counting** queries can be answered in $O(\log n)$ time. Argue that your augmentation does not change the asympotic preprocessing time and the asymptotic space complexity.

3. **Range Tree Construction (10 points)**

   (a) Describe a recursive algorithm that constructs a 1D range tree for a **sorted** set of n numbers in $O(n)$ time.

   (b) Same as above, but for a set of n unsorted numbers. Your algorithm should run in $O(n \log n)$ time.

   (c) Describe a recursive algorithm that constructs a 2D range tree for a set of n two-dimensional points in $O(n \log n)$ time.
   (Hint: Use a bottom-up approach, and the merge-routine from mergesort.)

4. **Smallest Rectangle Queries (10 points)**
   Let P be a set of n points in the plane; you may assume that they are in general position. Devise a data structure of size $O(n \log n)$ to answer queries of the following form in $O(\log^2 n)$ time:

   > Given a vertical line segment s and an integer k, find the smallest rectangle that has s as its left side and which contains at least k points. If no such rectangle exists then indicate this.

   (Hint: Use 2D range trees with fractional cascading. Where does the extra log-factor come from in the query time?)