CMPS 3120 Computational Geometry – Spring 13

1/24/13

1. Homework

Due **2/7/13** before class

1. **Data structures for convex polygons (6 points)**
   Describe at least two different data structures to preprocess a convex n-gon P in the plane, such that given a query point q it can be answered in $O(\log n)$ time whether $q \in P$ or not. What are the preprocessing time and the space requirements for your data structures? Try to make the preprocessing and space requirement of at least one of them as efficient as possible.

2. **Diameter (10 points)**
   Let the *diameter* of a point set $P = \{p_1, \ldots, p_n\} \in \mathbb{R}^2$ be the largest distance between any two points of P.

   (a) Prove that the diameter of P is achieved by two hull vertices.

   (b) Describe an algorithm that, given $CH(P)$, computes the diameter of P in linear time. You may research an algorithm for it on the web, but in that case you need to cite the source you were using and the explanation of the algorithm needs to be in your own words. As usual, also analyze the runtime.

3. **Lower bounds (9 points)**
   Consider the following problems:

   SORTING: Given a set $X = \{x_1, \ldots, x_n\}$ of n numbers, output the same numbers in increasing order.

   ELEMENT UNIQUENESS: Given a set $X = \{x_1, \ldots, x_n\}$ of n numbers, are there i, j, with $i \neq j$, such that $x_i = x_j$?

   CLOSEST PAIR: Given a point set $P = \{p_1, \ldots, p_n\} \in \mathbb{R}^2$, output the closest pair of points in P.

   DIAMETER: Given a point set $P = \{p_1, \ldots, p_n\} \in \mathbb{R}^2$. Compute the diameter of P.

   (a) Prove a lower bound of $\Omega(n \log n)$ for SORTING, by reducing from ELEMENT UNIQUENESS.

   (b) Prove a lower bound of $\Omega(n \log n)$ for CLOSEST PAIR by reducing from an appropriate problem.

   (c) Prove a lower bound of $\Omega(n \log n)$ for DIAMETER by reducing from an appropriate problem.

FLIP OVER TO BACK PAGE $\Longrightarrow$

4. **Visible segments sweep (10 points)**

   Let S be a set of n disjoint line segments in the plane, and let p be a point not on any of the line segments of S. We wish to determine all line segments of S that P can see, i.e., all line segments of S that contain some point q so that the open segment $\overset{\vdash\dashv}{pq}$ does not intersect any line segment of S.

   Give an $O(n \log n)$ time algorithm for this problem that uses a rotating half-line with its endpoint at p.