CMPS 2200 Introduction to Algorithms – Fall 12

11/12/12

8. Homework

Due **11/20/12** in the lab

1. **Red-black tree rotations (6 points)**
   Find a sequence of numbers which, when incrementally inserted into a red-black tree, causes the following sequence of rotations:

   right, left, right.

   You may start with an initially non-empty tree, and you may insert numbers that do not cause any rotations. But there should not be any additional rotations performed.

   Draw the sequence of trees that you obtain after each insertion. For each such tree indicate the node that violates the red-black tree condition, indicate the nodes that participate in the rotation, the type of the rotation, and the subtrees that correspond to each other before and after the rotation.
   Hint: Use a red-black tree demo from the web.

2. **Legal red-black trees (6 points)**
   Describe all legal red-black trees that store all the keys $1, 2, 3, 4, 5$.

3. **Number of keys in a B-tree (5 points)**

   Suppose you have a B-tree of minimum degree k and height h. What is the largest number of keys that can be stored in such a B-tree? *(Hint: Your answer should depend on k and h. This is similar to slide 12 in the B-tree slides.)*

4. **Red-black trees and (2,3,4)-trees (8 points)**

   A $(2, 3, 4)$-tree is a B-tree with minimum degree $k = 2$. As we have seen on slides 11–16 of the red-black tree slides in class, a red-black tree can be converted into a $(2, 3, 4)$-tree by merging the red nodes into their black parent nodes.

   (a) Show how the $(2, 3, 4)$-tree below can be converted into a valid red-black tree.
   (b) Describe how an arbitrary $(2, 3, 4)$-tree can be converted into a valid red-black tree. Justify why your conversion yields a valid red-black tree.
   (c) Insert the number 18 into the $(2, 3, 4)$-tree below and show the resulting tree.
   (d) How do the B-tree operations for inserting 18 correspond to operations in the corresponding red-black tree?