CMPS 2200 Introduction to Algorithms – Fall 12

10/15/12

5. Homework

Due **10/23/12** in the lab

1. **Adjacency lists vs. adjacency matrix (7 points)**

   (a) (3 points) Give pseudo-code to convert a graph given in adjacency lists representation to its adjacency matrix representation. What is the runtime?

   (b) (4 points) Both DFS and BFS include the following `for`-loop referring to vertices `v` and `w`:

   ```
   for each w adjacent to v do{
      // some statement
   }
   ```

   Give pseudo-code that implements this loop using (i) adjacency lists and (ii) an adjacency matrix. Analyze the runtime for both (assume that the statement inside the loop takes O(1) time).

2. **DFS edges (9 points)**

   Make a 3-by-3 chart with row and column labels "unvisited, visited, finished". In each cell (i, j) indicate whether, at any point during a DFS of a **directed** graph, there can be an edge from a vertex with property i to a vertex with property j. For each possible edge, indicate what edge type it can be (there can be multiple per cell). Explain your answers shortly. *(Hint: Refer to slide 42 of the graph slides.)*

3. **Dijkstra and negative edge weights (4 points)**

   Give an example of a directed connected graph with negative edge weights, but without a negative weight cycle, for which Dijkstra's algorithm produces incorrect answers. Justify your answer.

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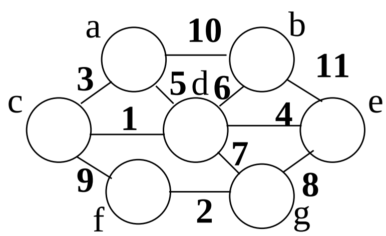
4. **Dijkstra (7 points)**
Run Dijkstra's algorithm on the graph below, with start/source vertex a. (Assume that each undirected graph edge $\{u, v\}$ is represented using two directed edges (u, v) and (v, u) with the same weight.)

(a) Show all the different stages of the algorithm, including vertex weights, vertices in S, the vertex extracted from the priority queue, and the tree edges stored in the predecessor array.



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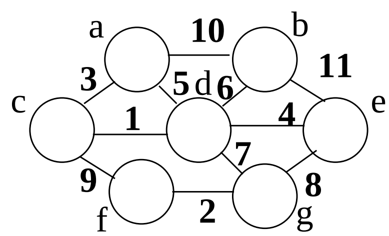

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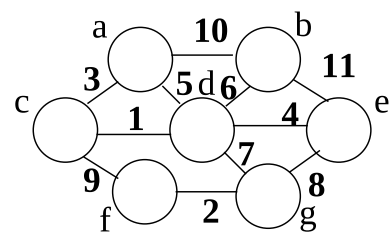

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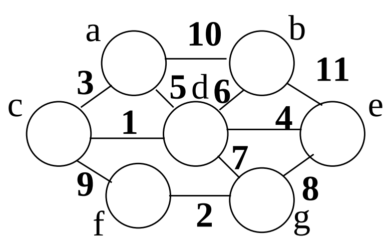

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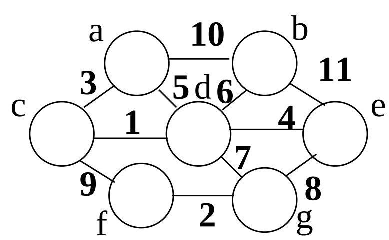

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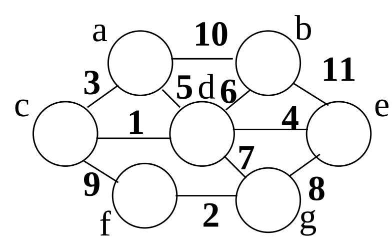

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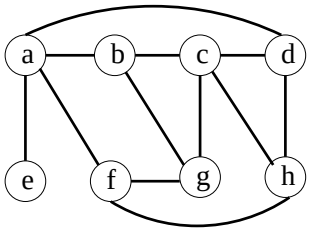
(b) List the shortest paths from a to all other vertices.

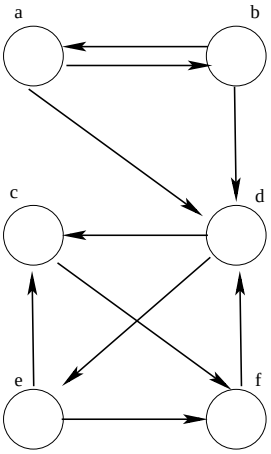
Practice Problems

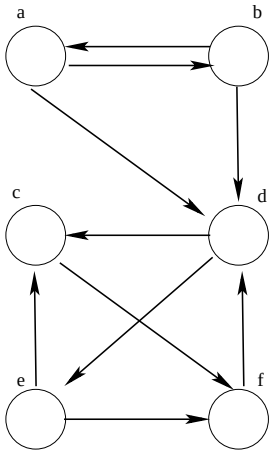
(Not required for homework credit.)



1. **Adjacency matrix and lists**
   Give the adjacency matrix representation and the adjacency lists representation for the graph above. Assume that vertices (e.g., in adjacency lists) are ordered alphabetically.

2. **DFS, BFS**
   Run BFS and DFS on the graph below, starting on vertex a. Write the visit times (and for DFS finish times) into the vertices. Assume that vertices are ordered alphabetically in the adjacency lists. For DFS also show the edge classifications.





3. **Dijkstra and negative edge weights**

   Give an example of a directed connected graph with a negative weight cycle for which Dijkstra's algorithm produces incorrect answers. Justify your answer.