CMPS 2200 Introduction to Algorithms – Fall 12

9/5/12

1. Homework

Due **9/18/12** in the lab

1. **Big-Oh ranking (10 points)**
   Rank the following ten functions by order of growth, i.e., find an arrangement $f_1, f_2, ...$ of the functions satisfying $f_1 \in O(f_2)$, $f_2 \in O(f_3)$,... . Partition your list into equivalence classes such that f and g are in the same class if and only if $f \in \Theta(g)$. For every two functions f_i, f_j that are adjacent in your ordering, prove shortly why $f_i \in O(f_j)$ holds. And if f and g are in the same class, prove that $f \in \Theta(g)$.

   $$\log n \ , \ \sqrt[3]{n} \ , \ 2^{2n} \ , \ \log\log n \ , \ n^2 \ , \ n \log n \ , \ 2^n \ , \ \sqrt{n} \ , \ n \ , \ 2^{n+1} \ ,$$

   Bear in mind that in some cases it might be useful to show $f(n) \in o(g(n))$, since $o(g(n)) \subset O(g(n))$. If you try to show that $f(n) \in o(g(n))$, then it might be useful to apply the rule of l'Hôpital which states that

   $$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{f'(n)}{g'(n)}$$

   if the limits exist; where f and g' are the derivatives of f and g, respectively.

2. **O, Ω, Θ (8 points)**

   Show using the definitions of big-Oh, Ω, and Θ:

   (a) (4 points) $7n^4 + 2n^2 - 8n \in \Theta(n^4)$

   (b) (4 points) If $f_1(n) \in O(g_1(n))$ and $f_2(n) \in O(g_2(n))$, prove that

   $$f_1(n) + f_2(n) \in O(\max(g_1(n), g_2(n))) \ .$$

   Is the same true for $O(\min(g_1(n), g_2(n)))$? Explain your answer.

3. **Code snippet (6 points)**

   Give the Θ-runtime for the code snippet below, depending on n. Justify your answer.

```
for(i=1; i<=n; i=i*3)
  for(j=n; j>=1; j=j/3)
      for(l=2; l<=n; l=l*l*l)
        print(" ");
```