Functional Programming III

Spring 2014

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Merge Sort

Suppose that we know how to merge two sorted lists. Then, we can sort recursively:

Merge Sort:

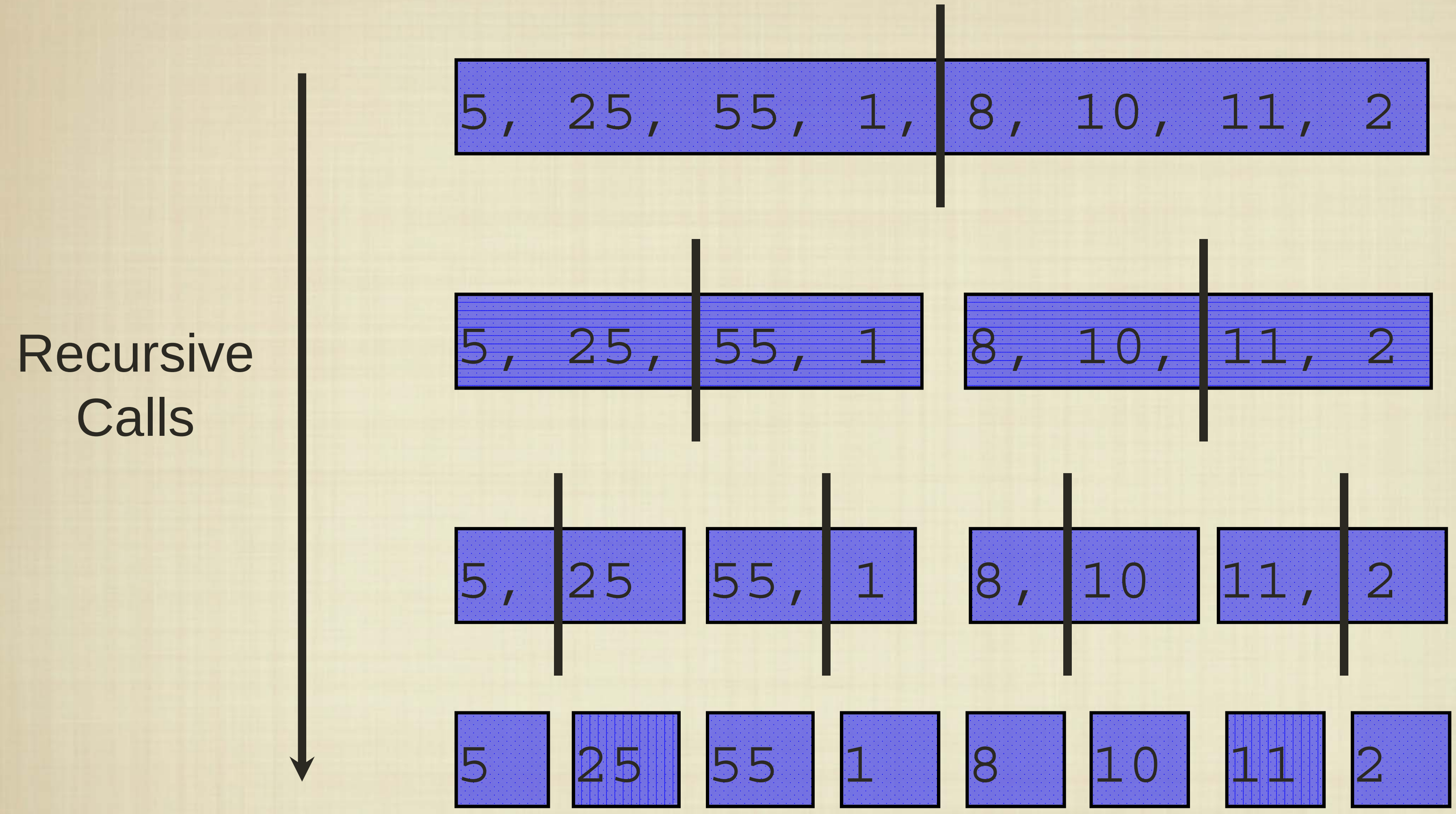
- 1. Split the given list into two equal parts.
- 2. Recursively sort each half.
- 3. Merge the sorted halves and return the result.

Merge Sort (Python)

<ins>Merge Sort:</ins>
1. Split the given list into two equal parts.
2. Recursively sort each half.
3. Merge the sorted halves and return the result.

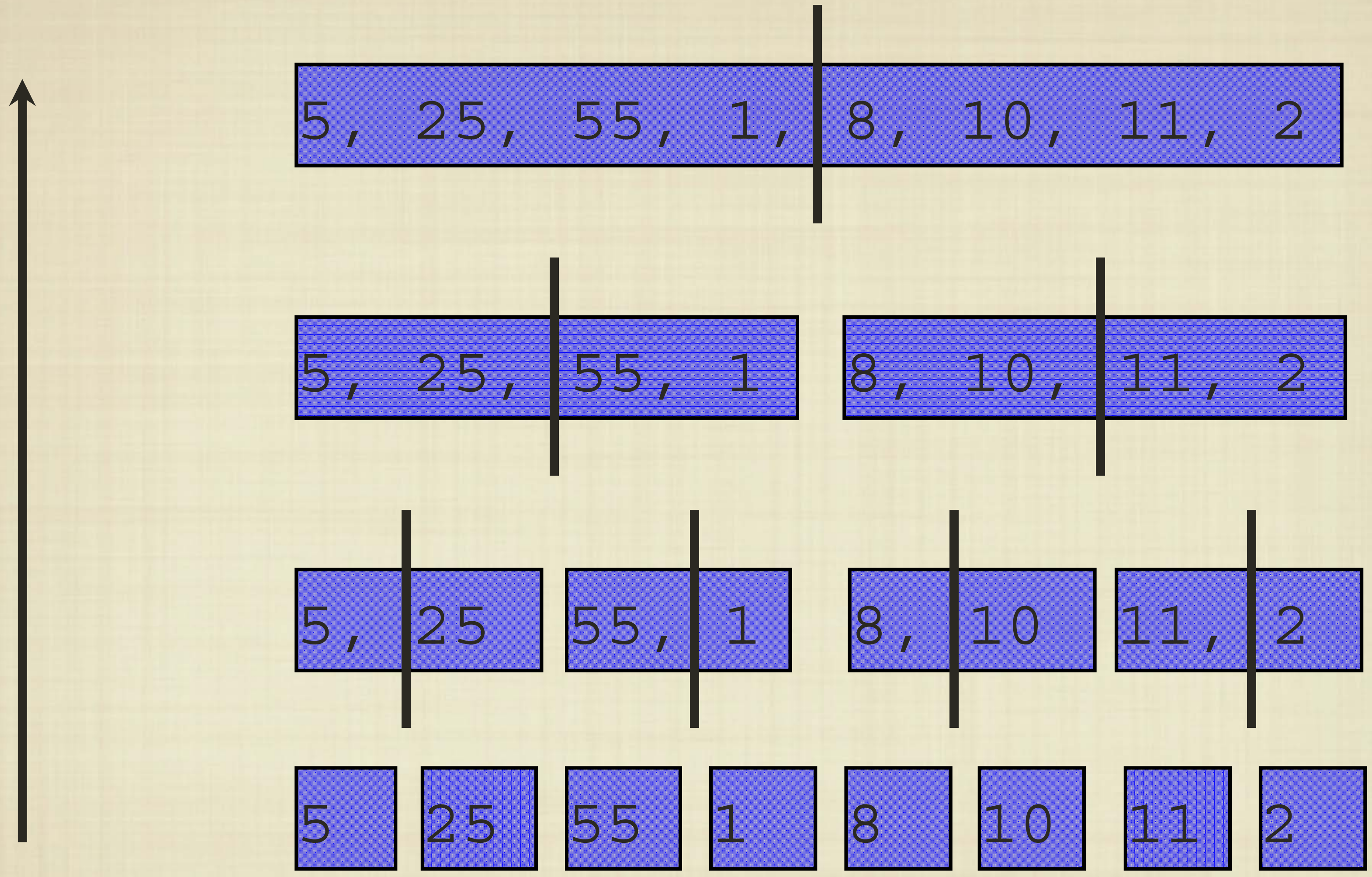
```
def merge_sort (L):
    n = len(L)
    #base case:
    if n<=1:
        return L
    #recursive case: Recursively sort each half
    A = merge_sort(L[:n/2])   # left half, L[0..n/2-1]
    B = merge_sort(L[n/2:])   # right half, L[n/2..n-1]
    # merge sorted halves:
    return merge(A,B)
```

Merge Sort



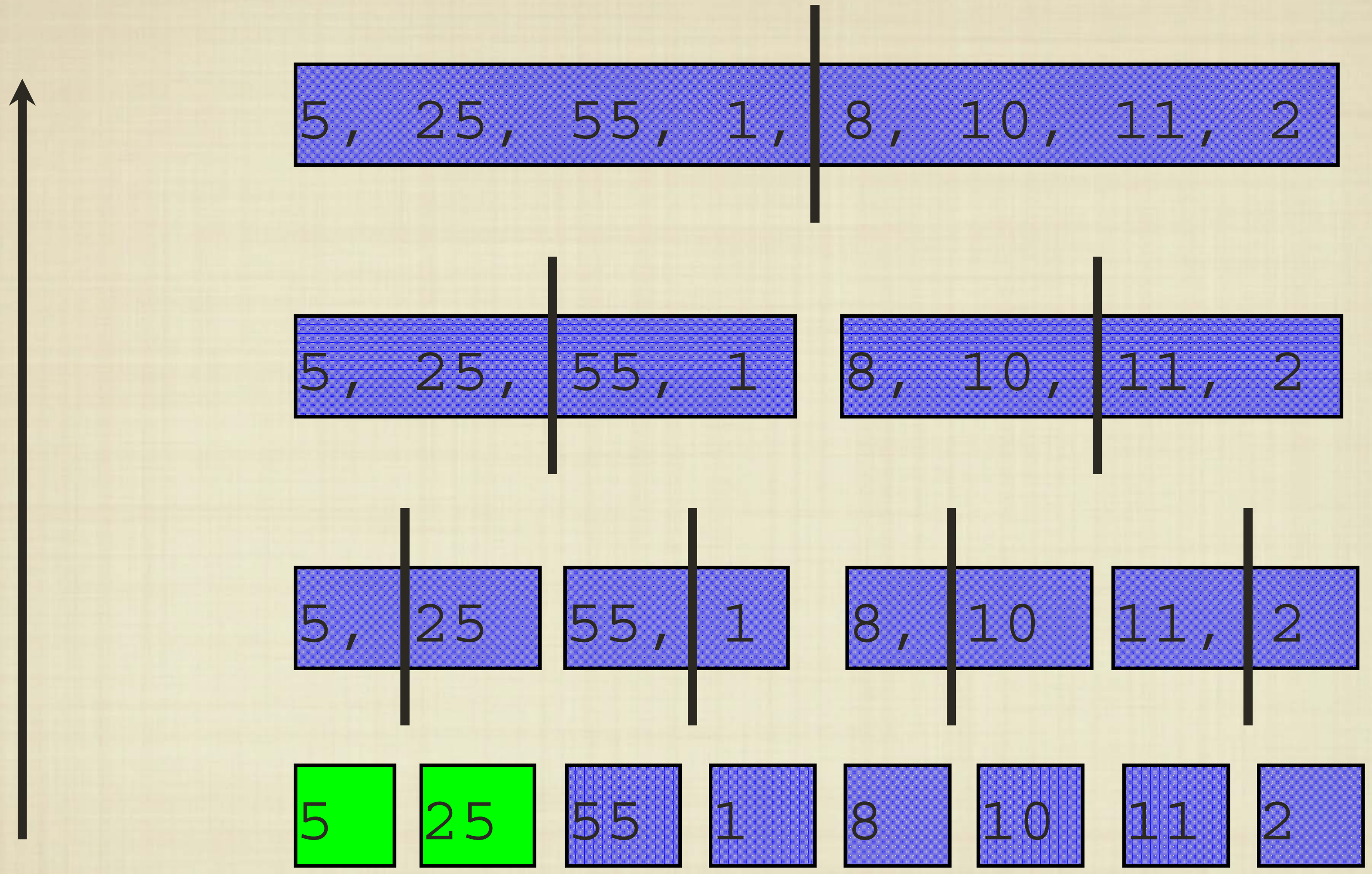
Actually, not a lot is happening in the recursive calls. So where is the sorting happening?

Merge Sort



The merge step is actually doing all of the work!

Merge Sort



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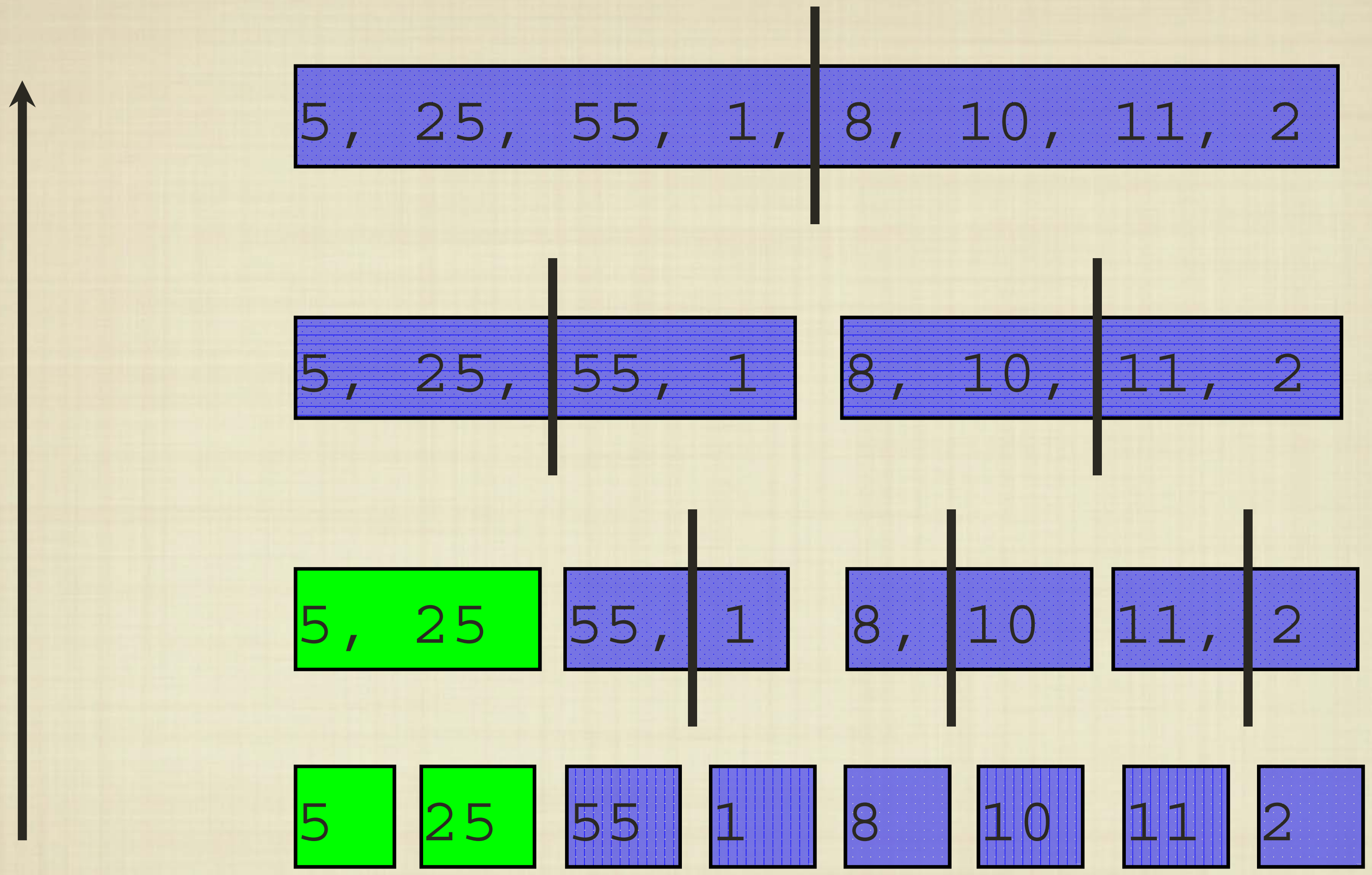

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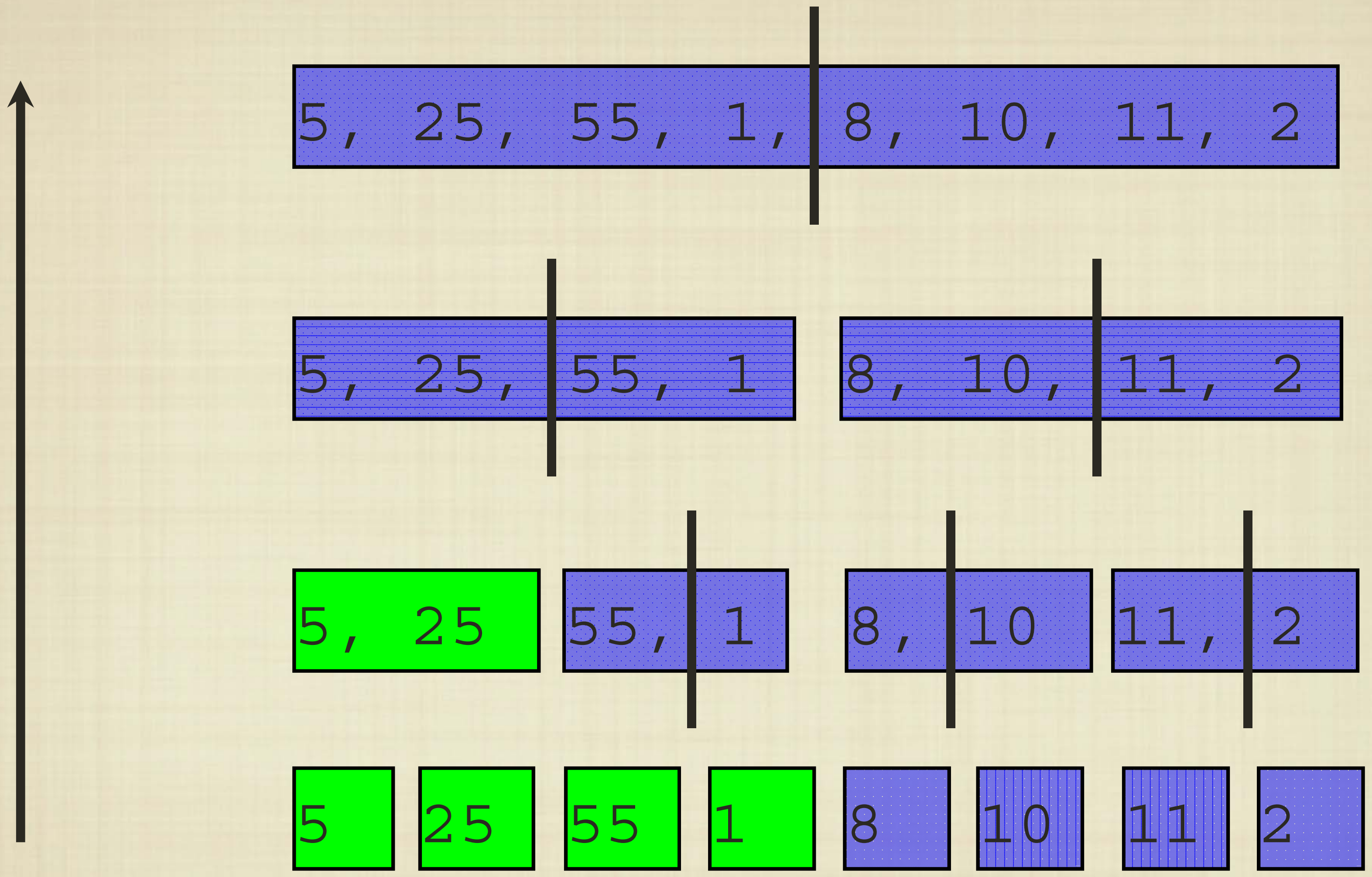

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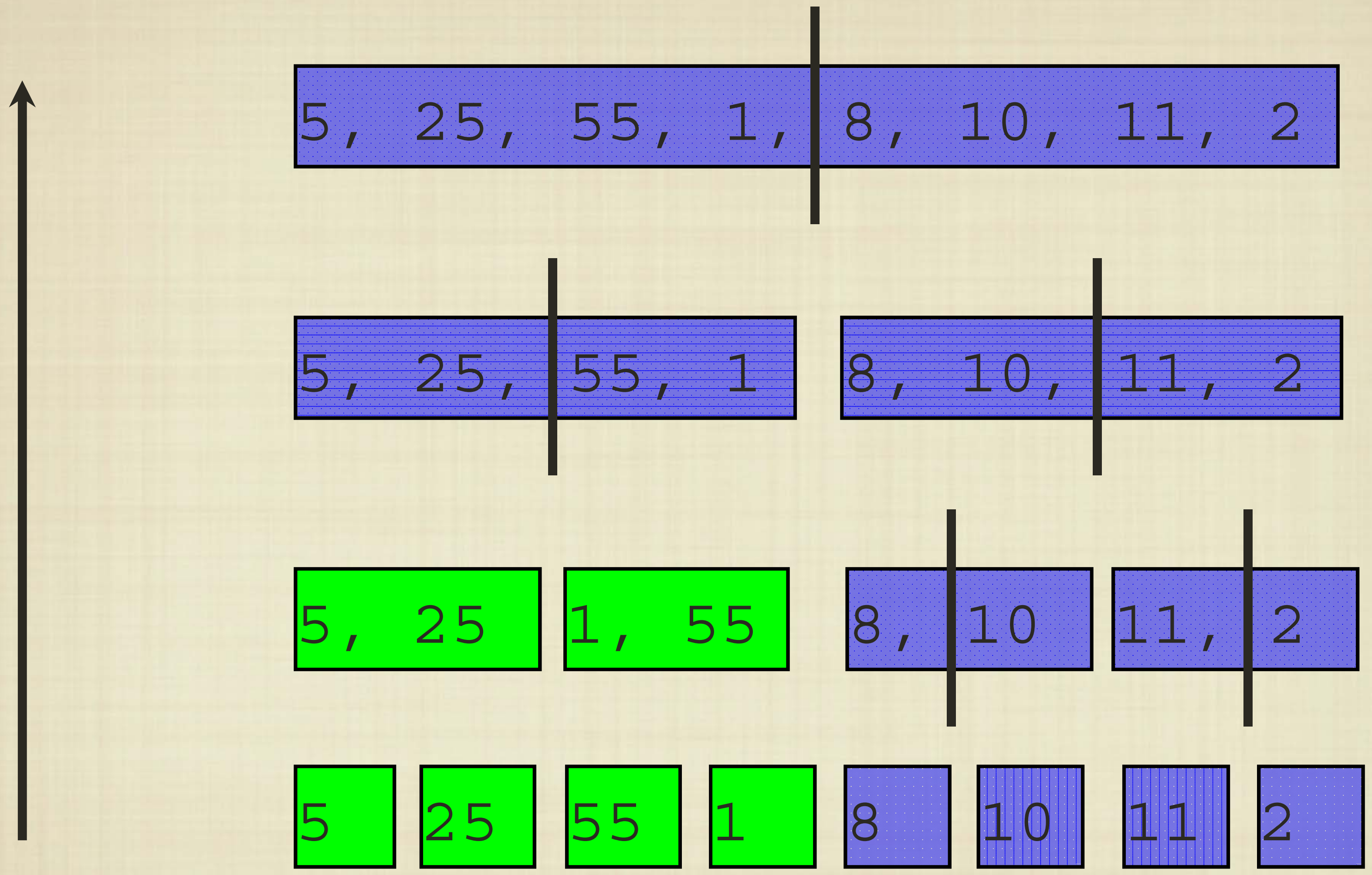

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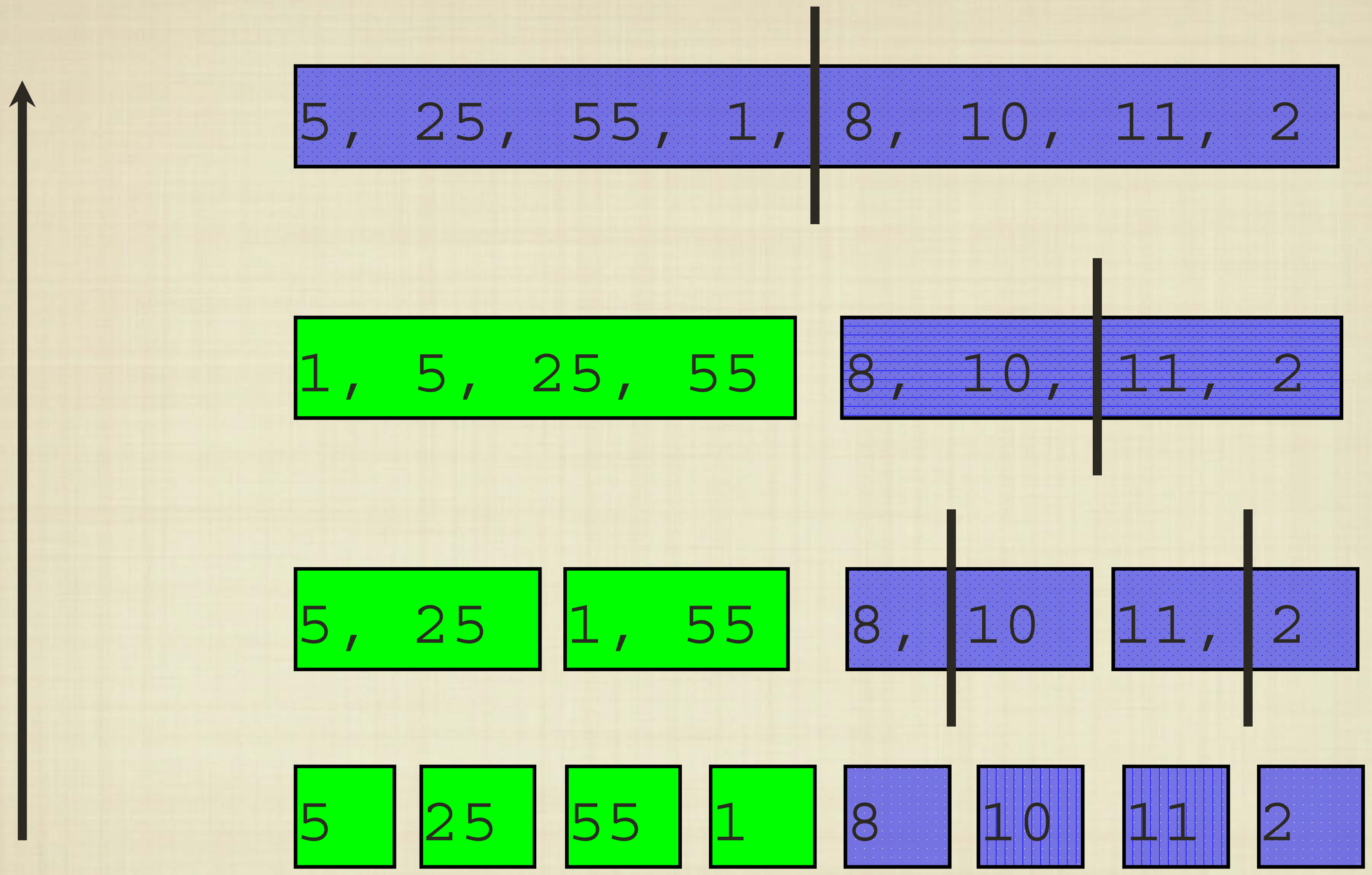

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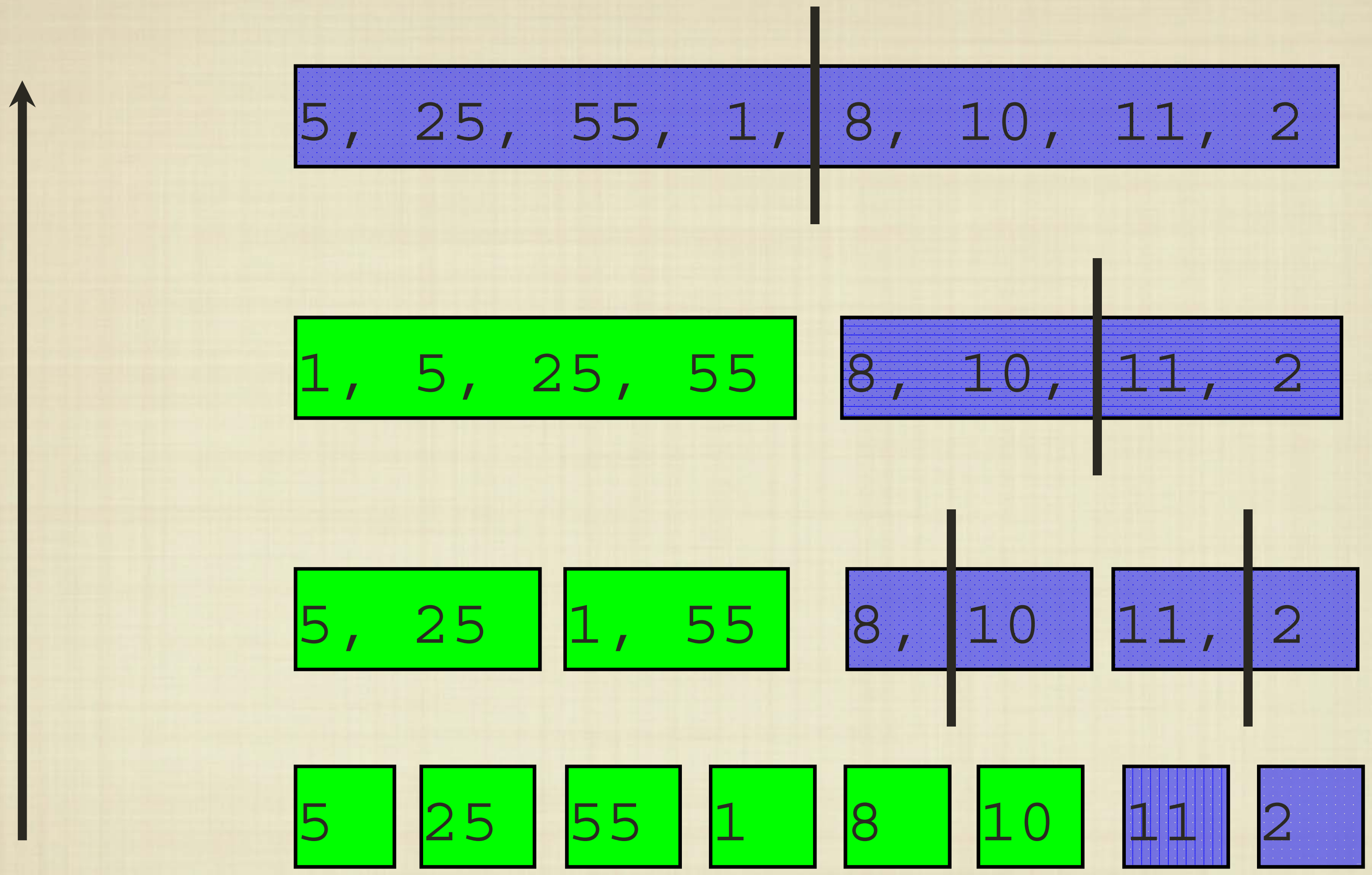

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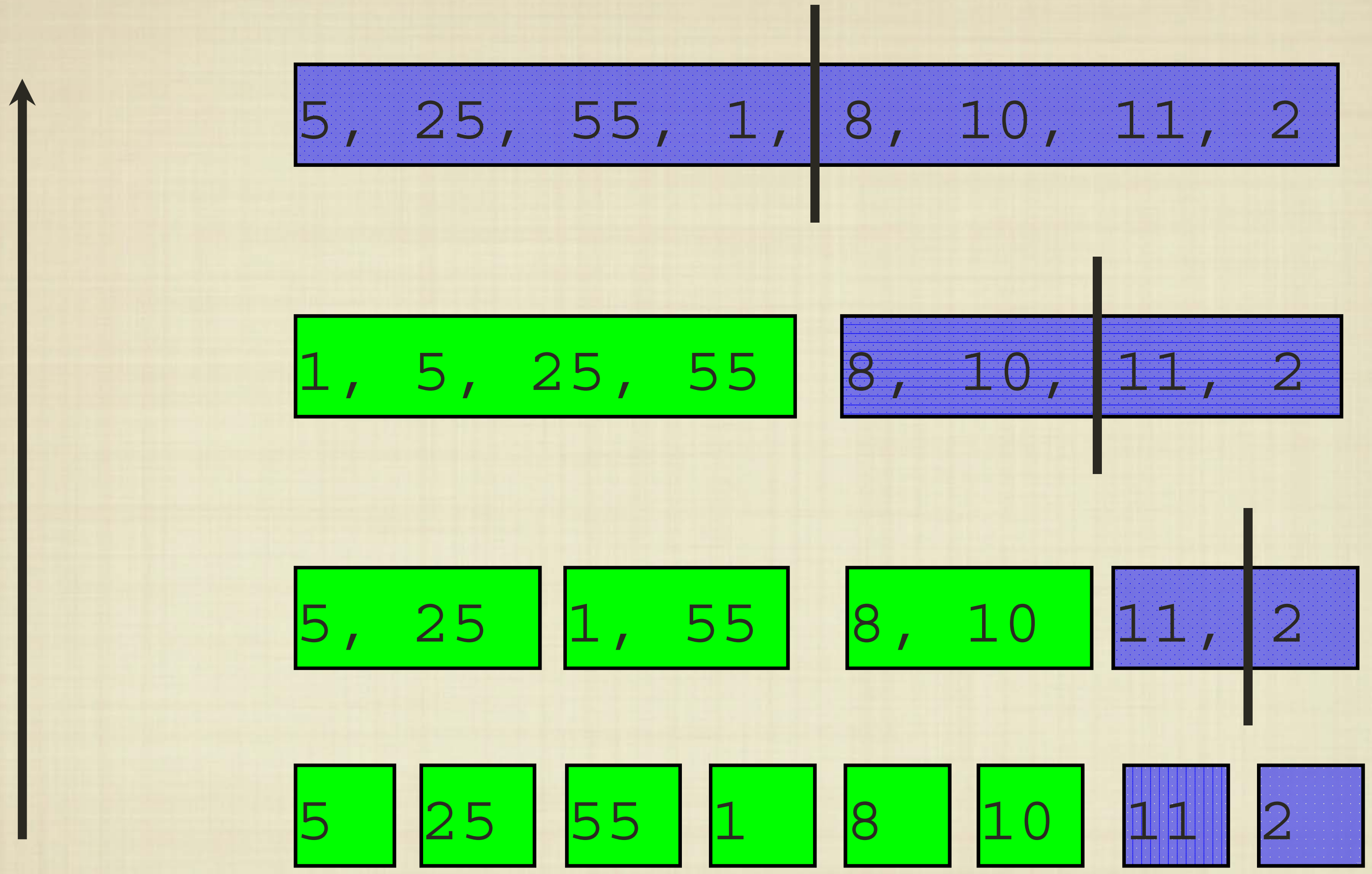

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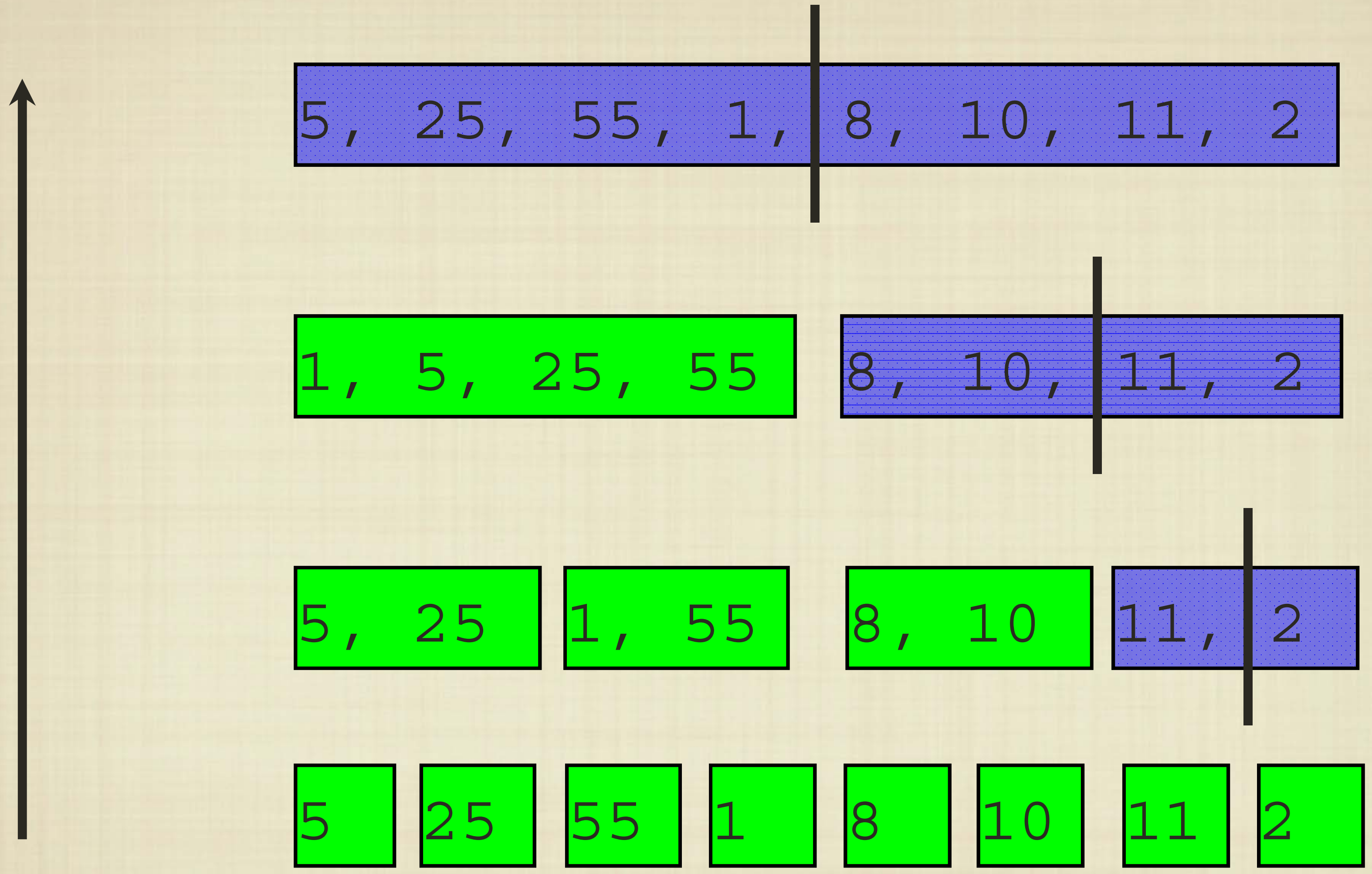

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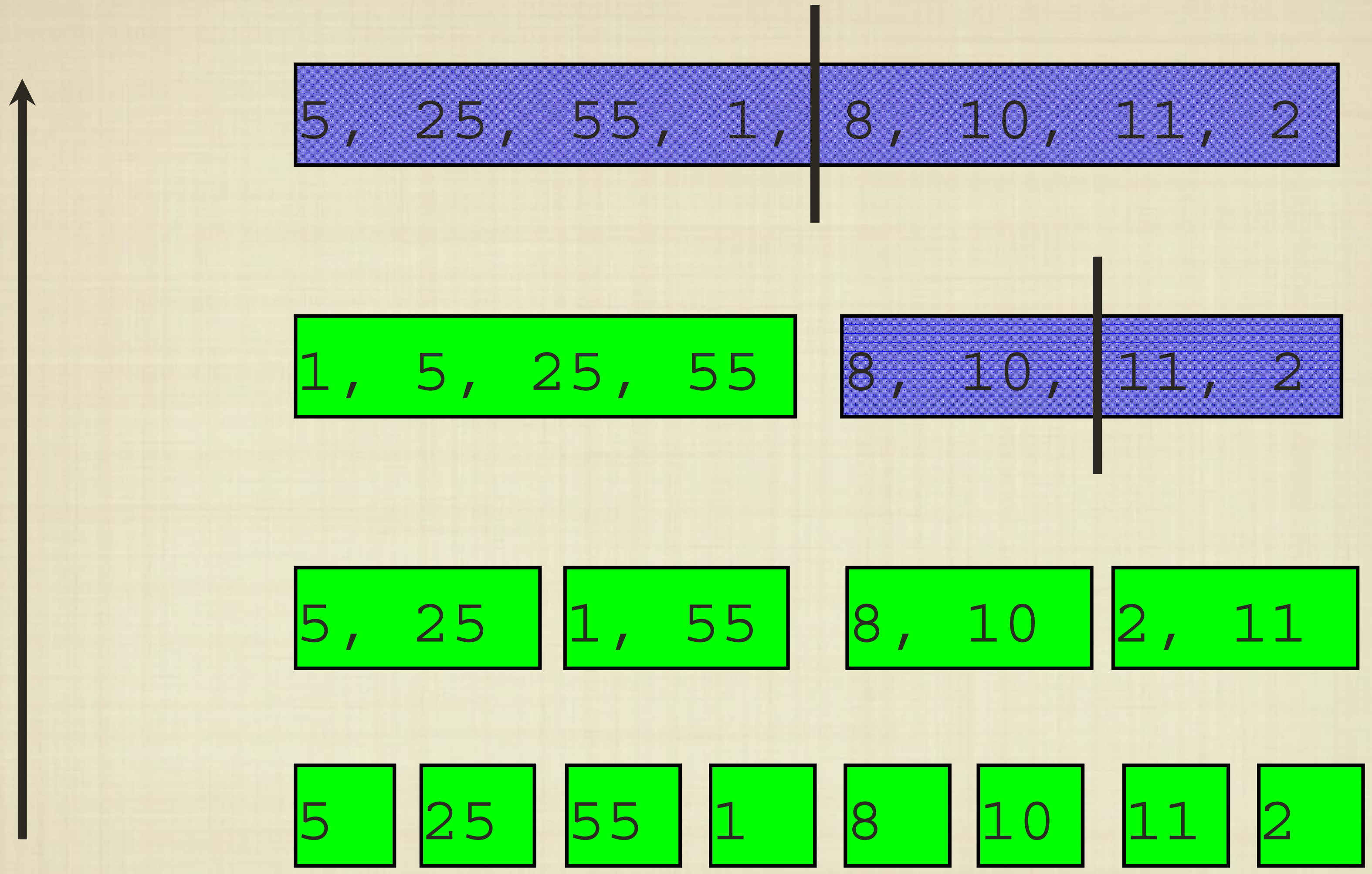

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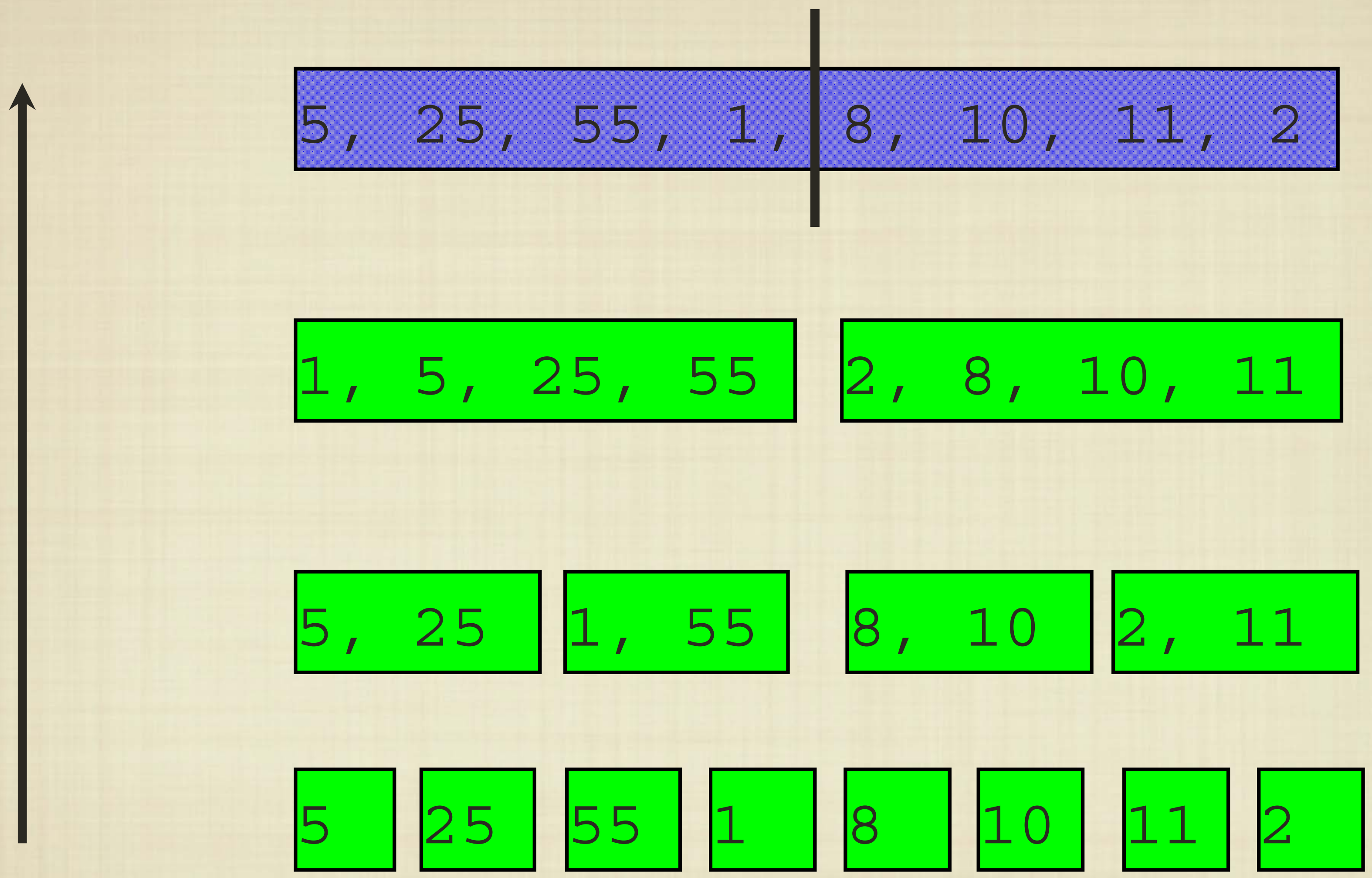

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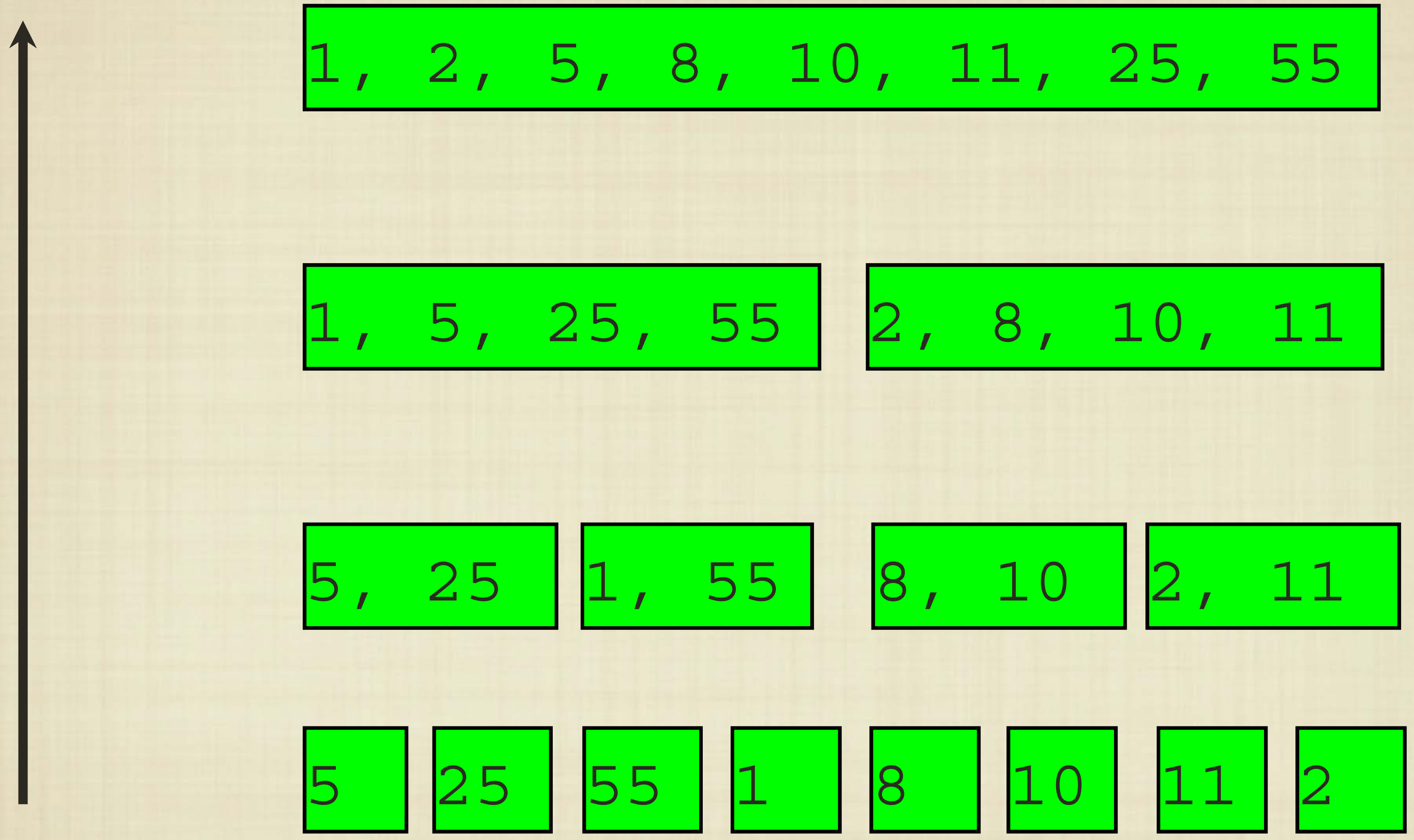

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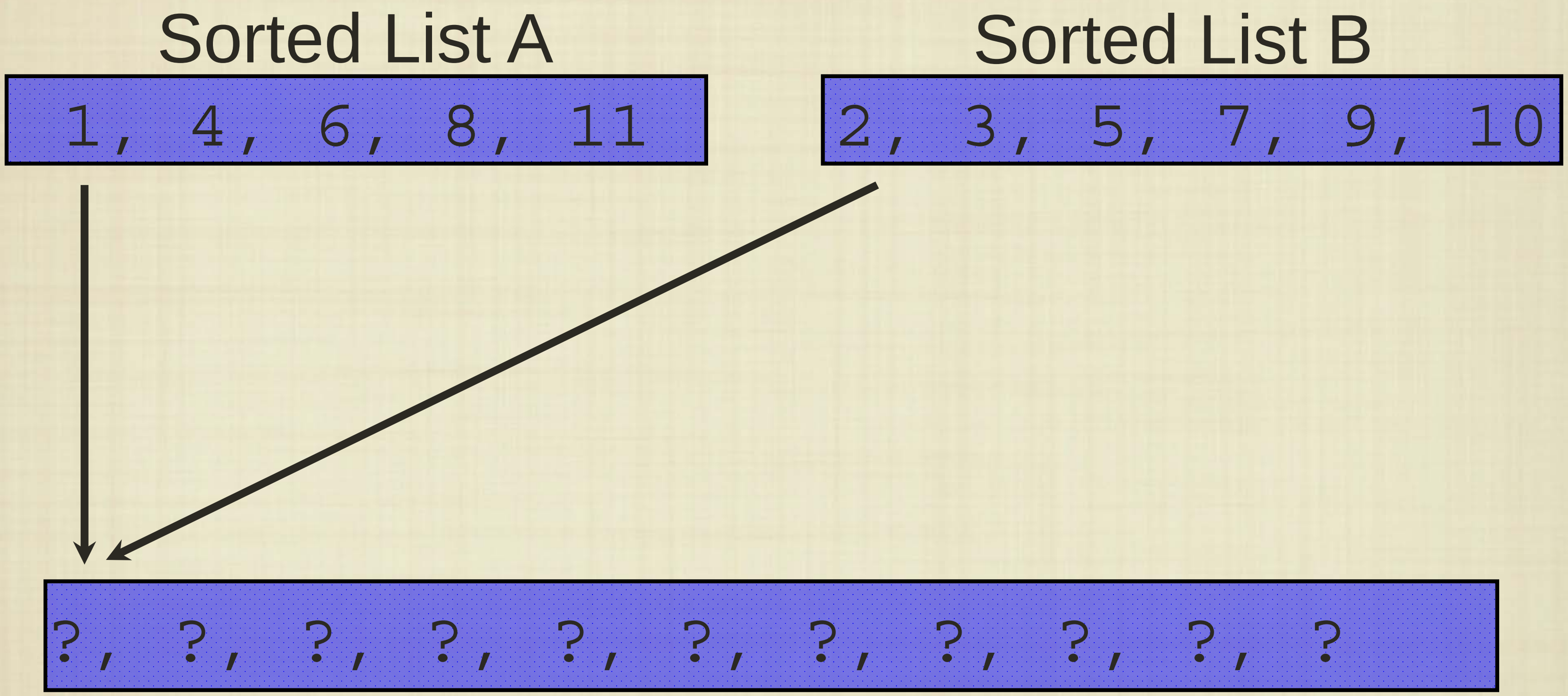
Merging Lists



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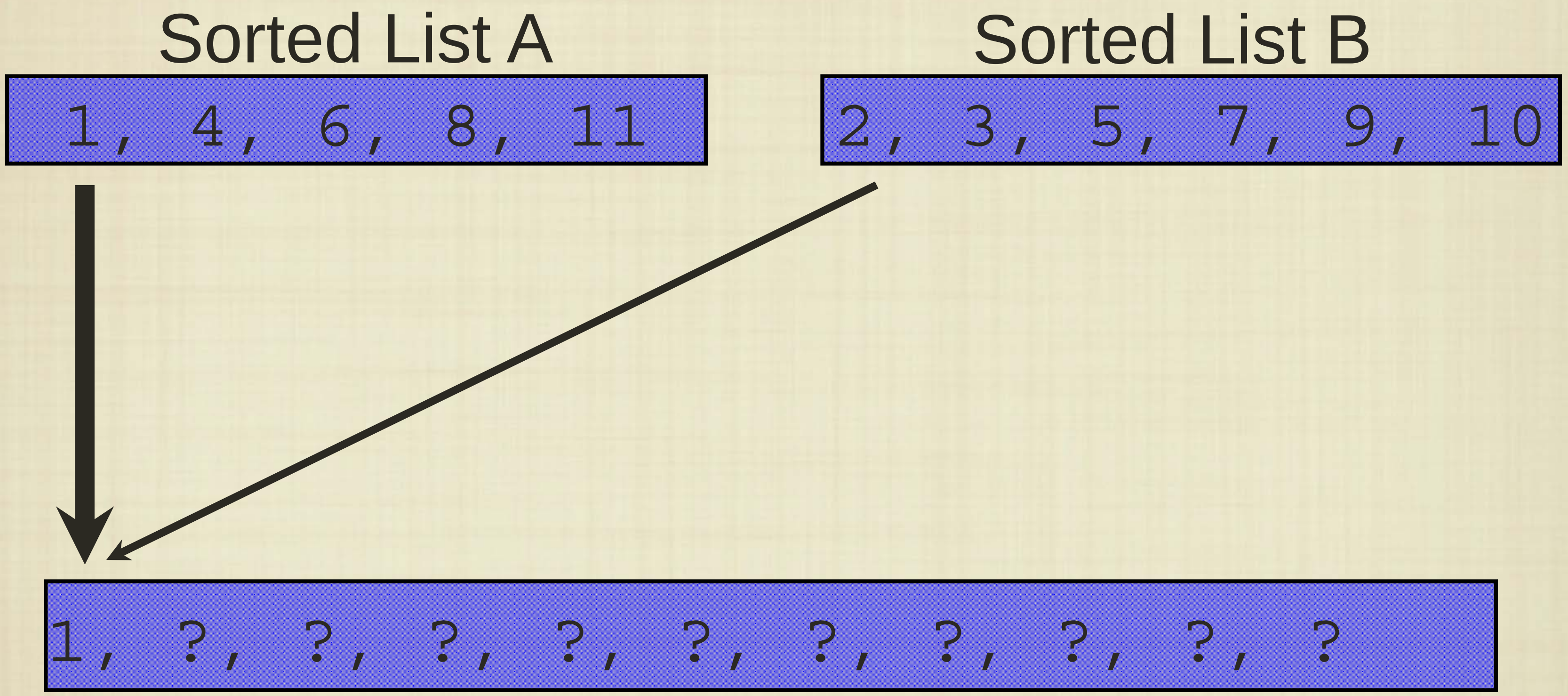

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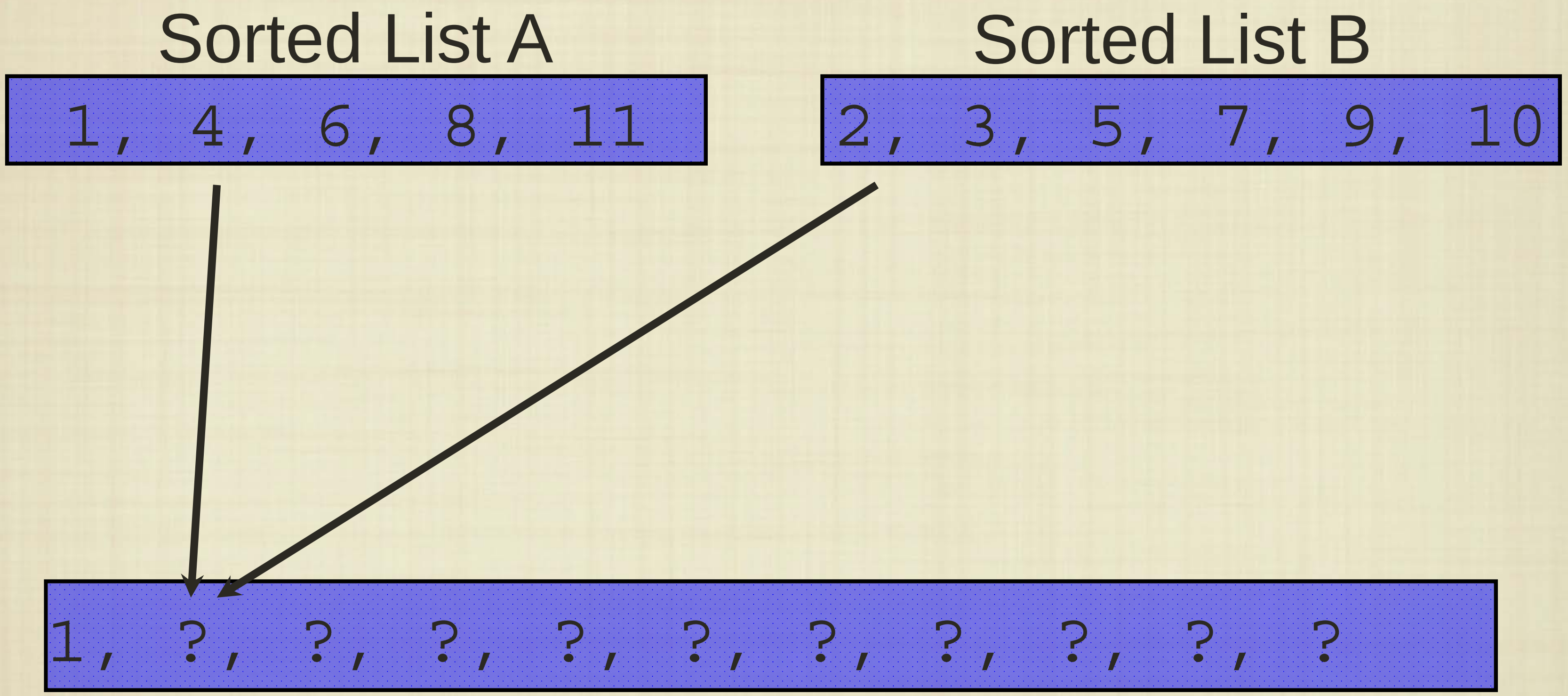

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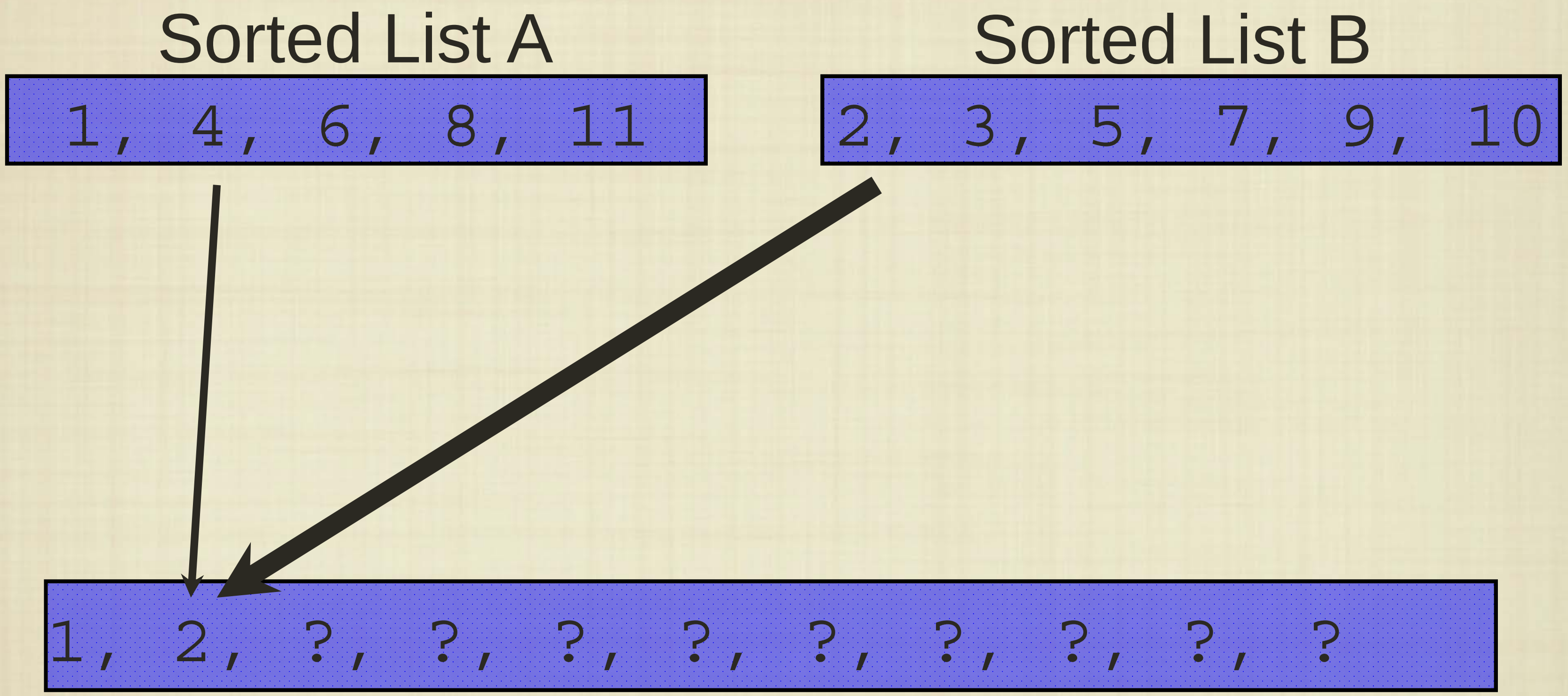

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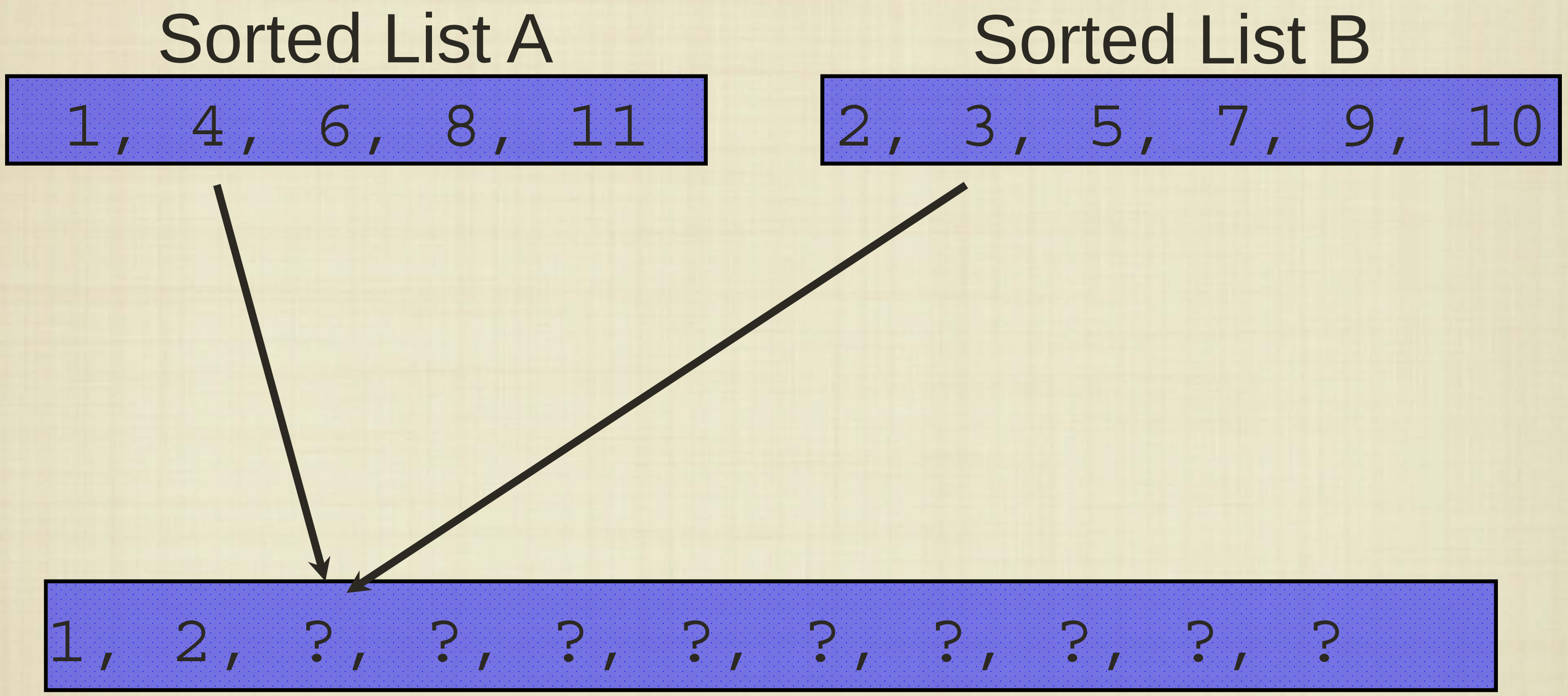

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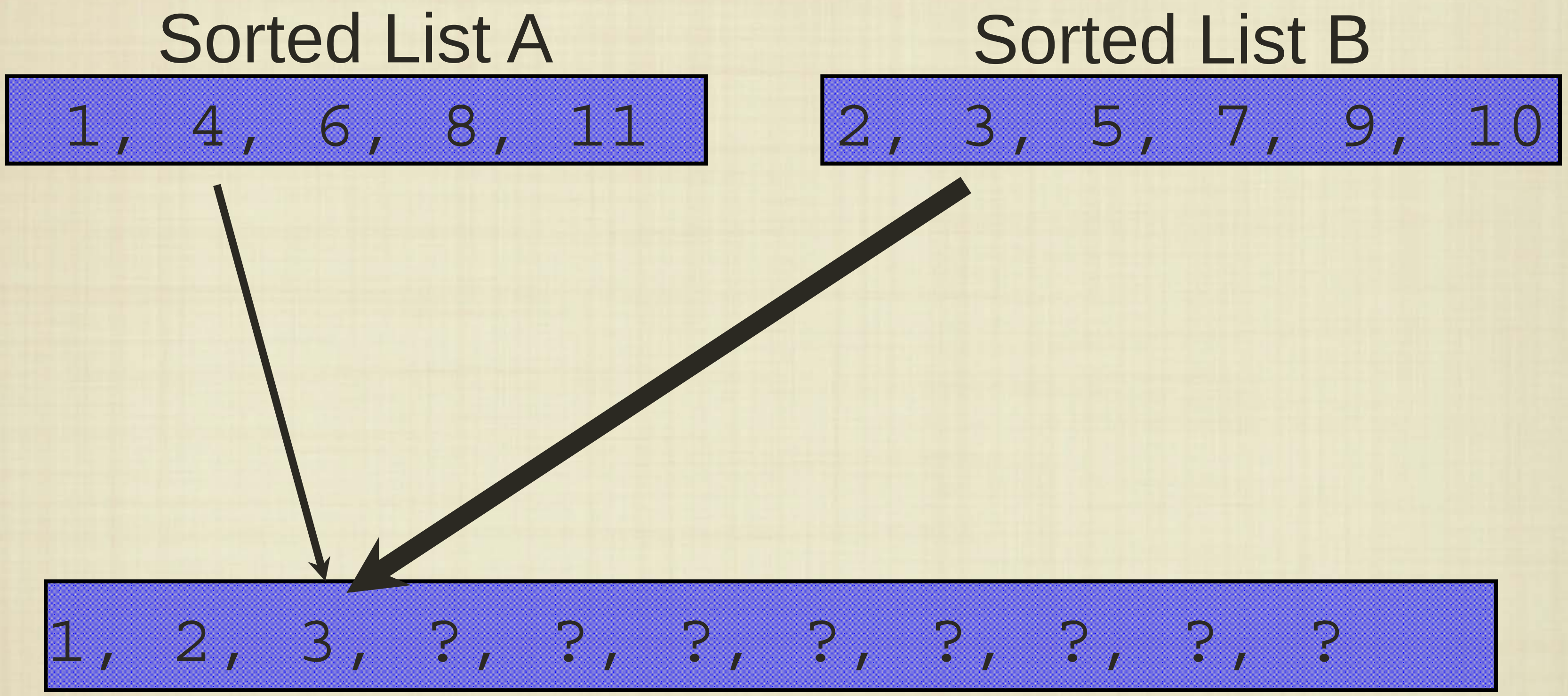

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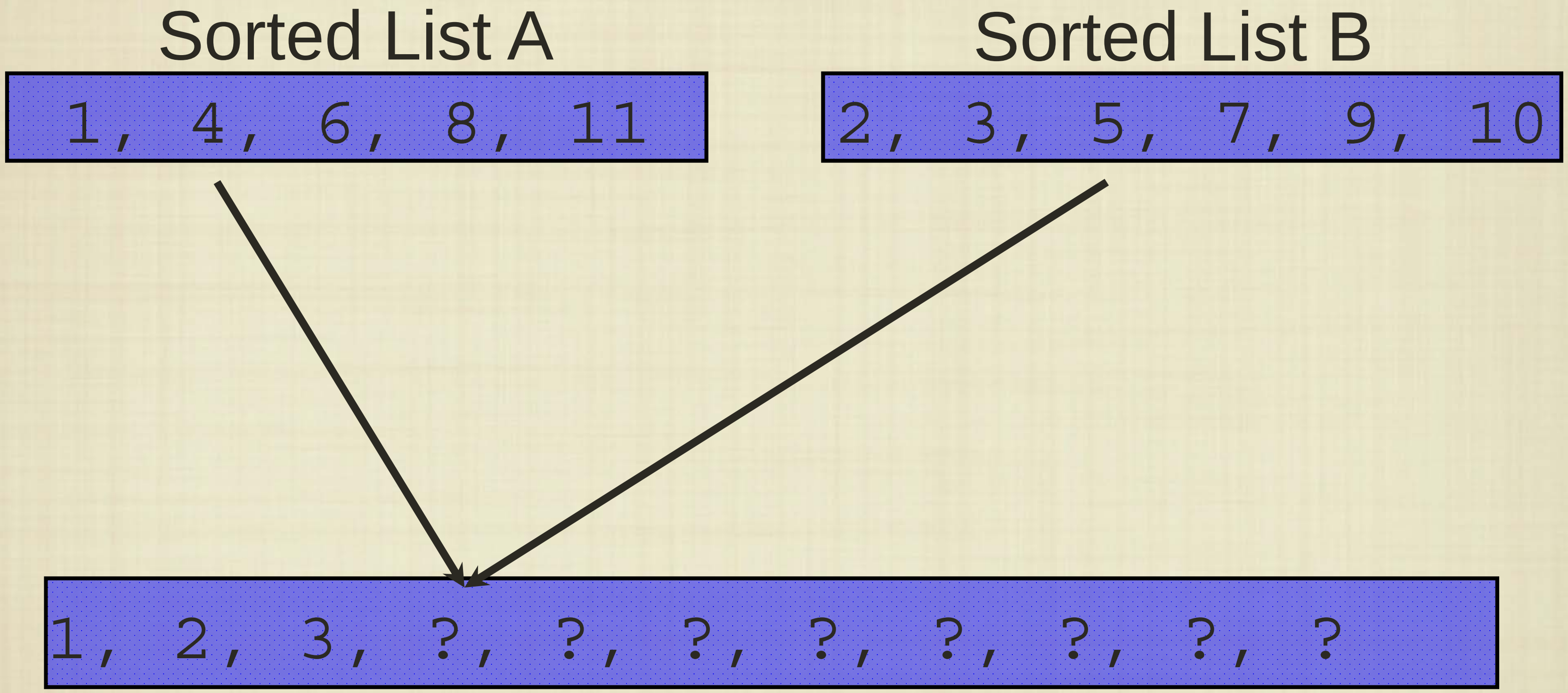

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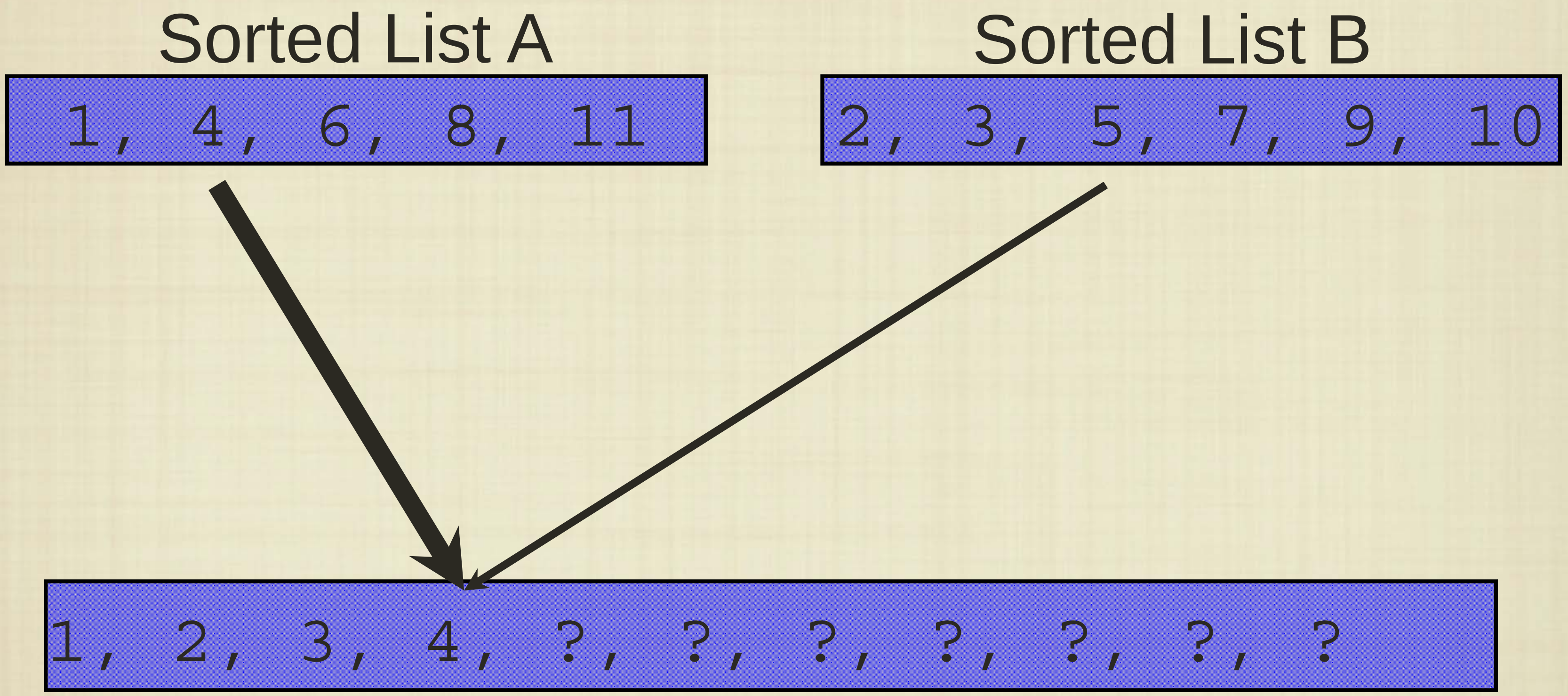

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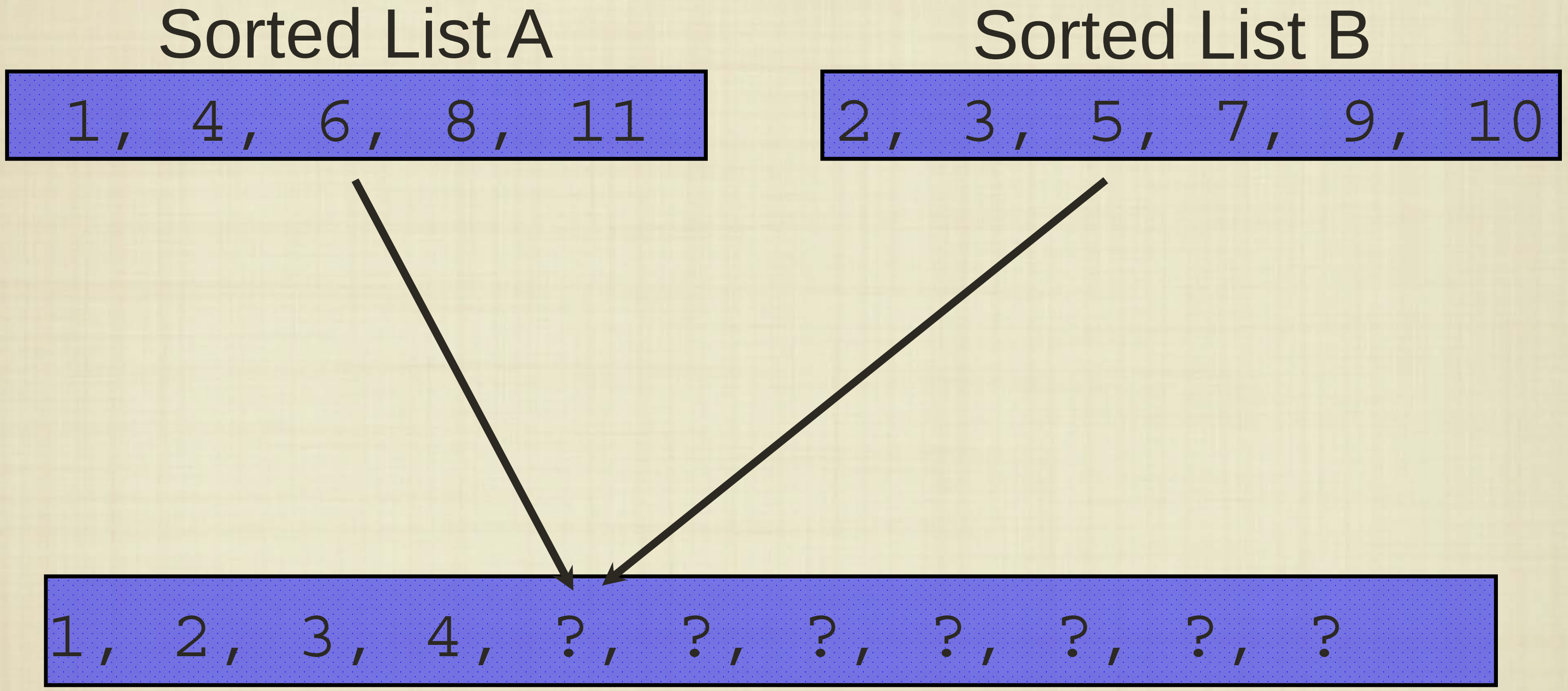

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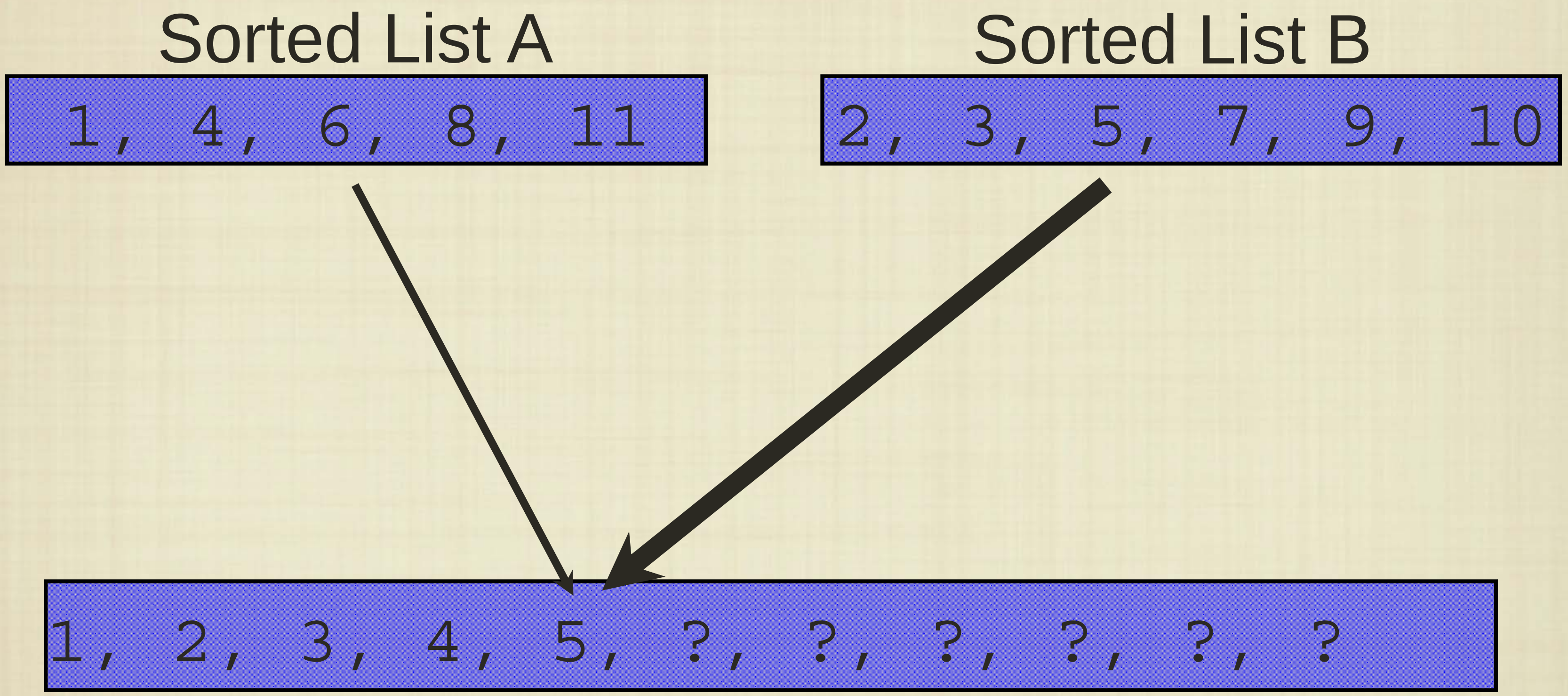

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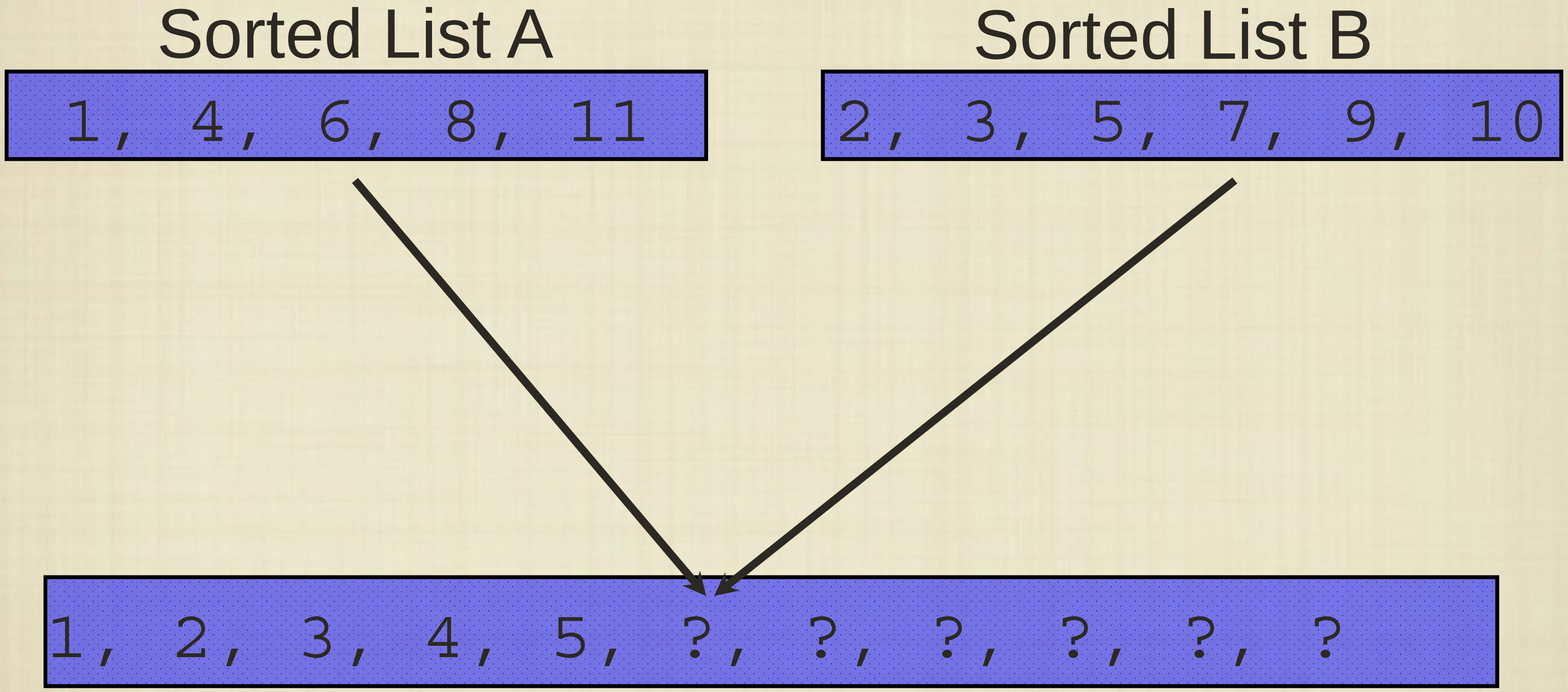

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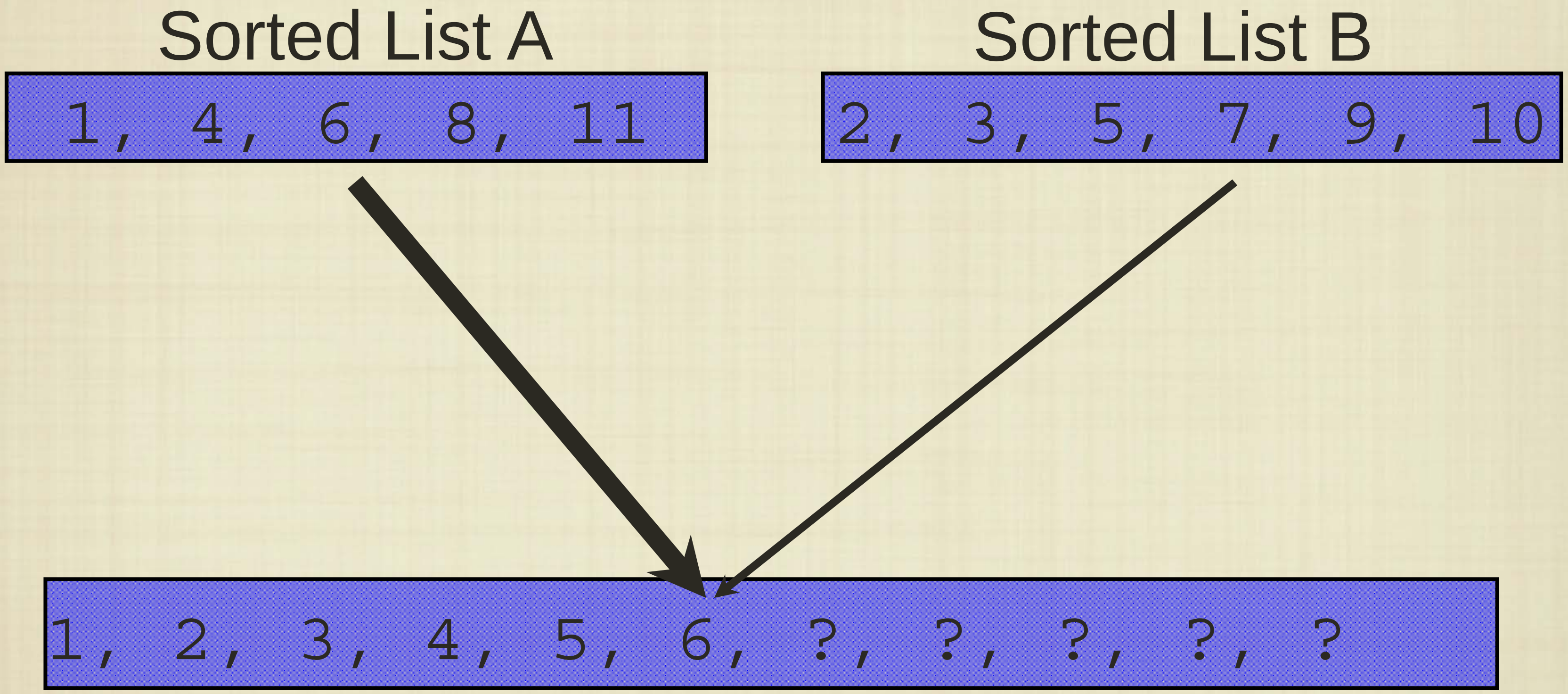
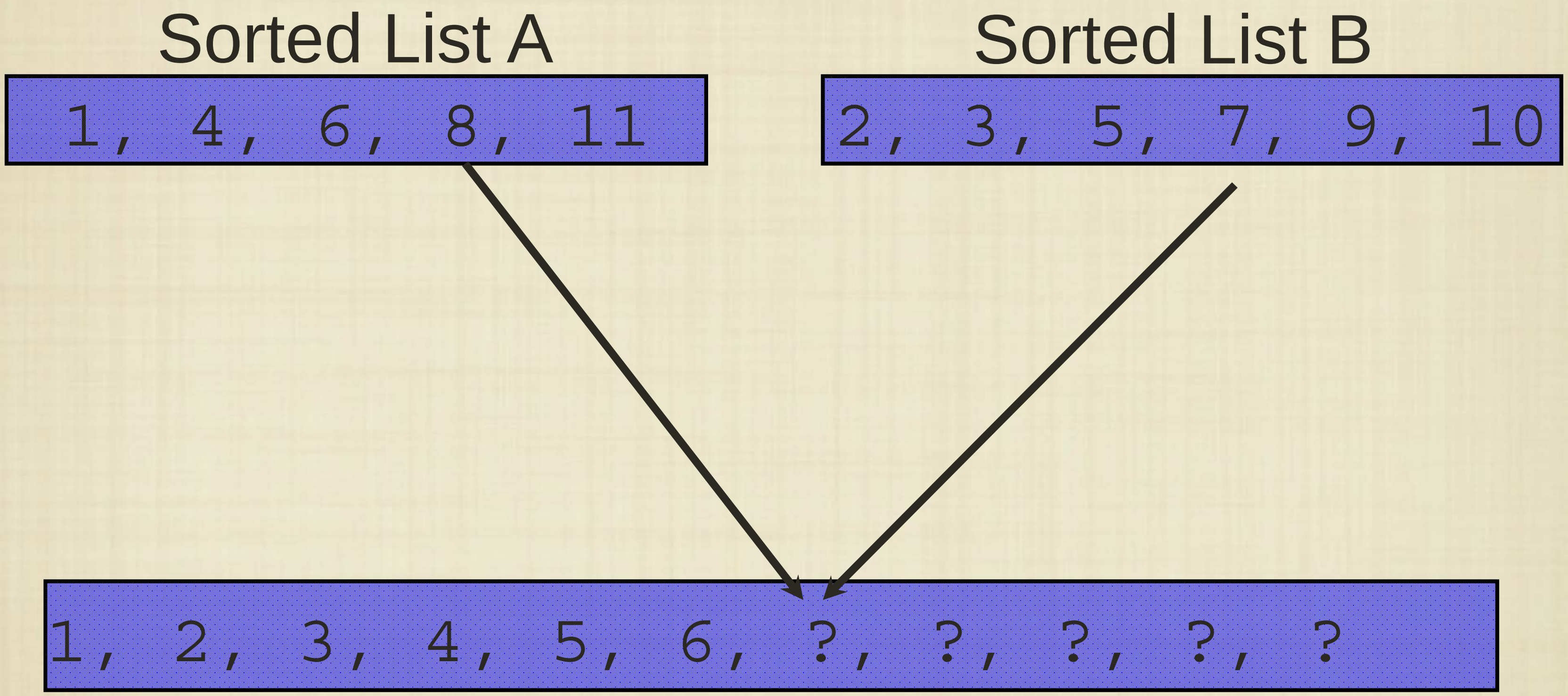


Merging Lists

- Suppose that we instead had a list that had two sorted halves. Could we do better?



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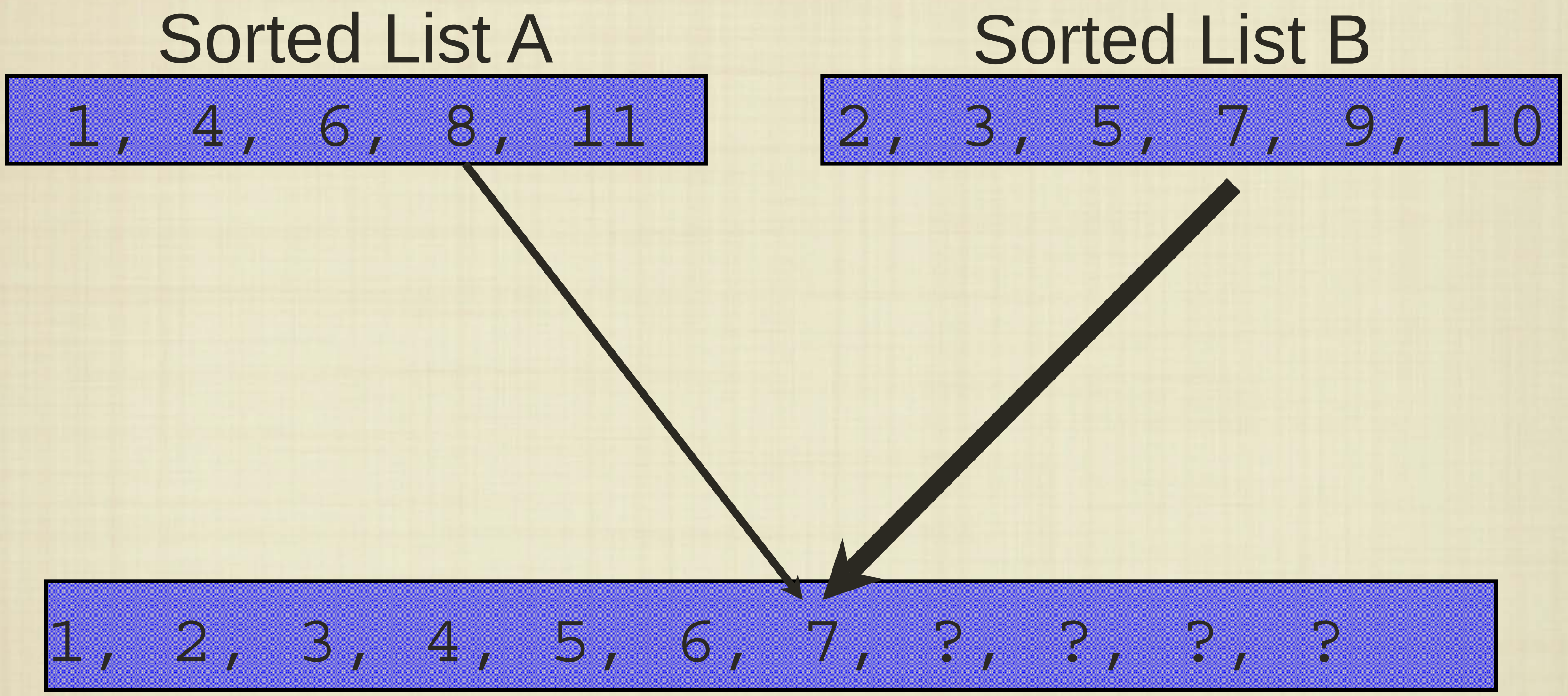

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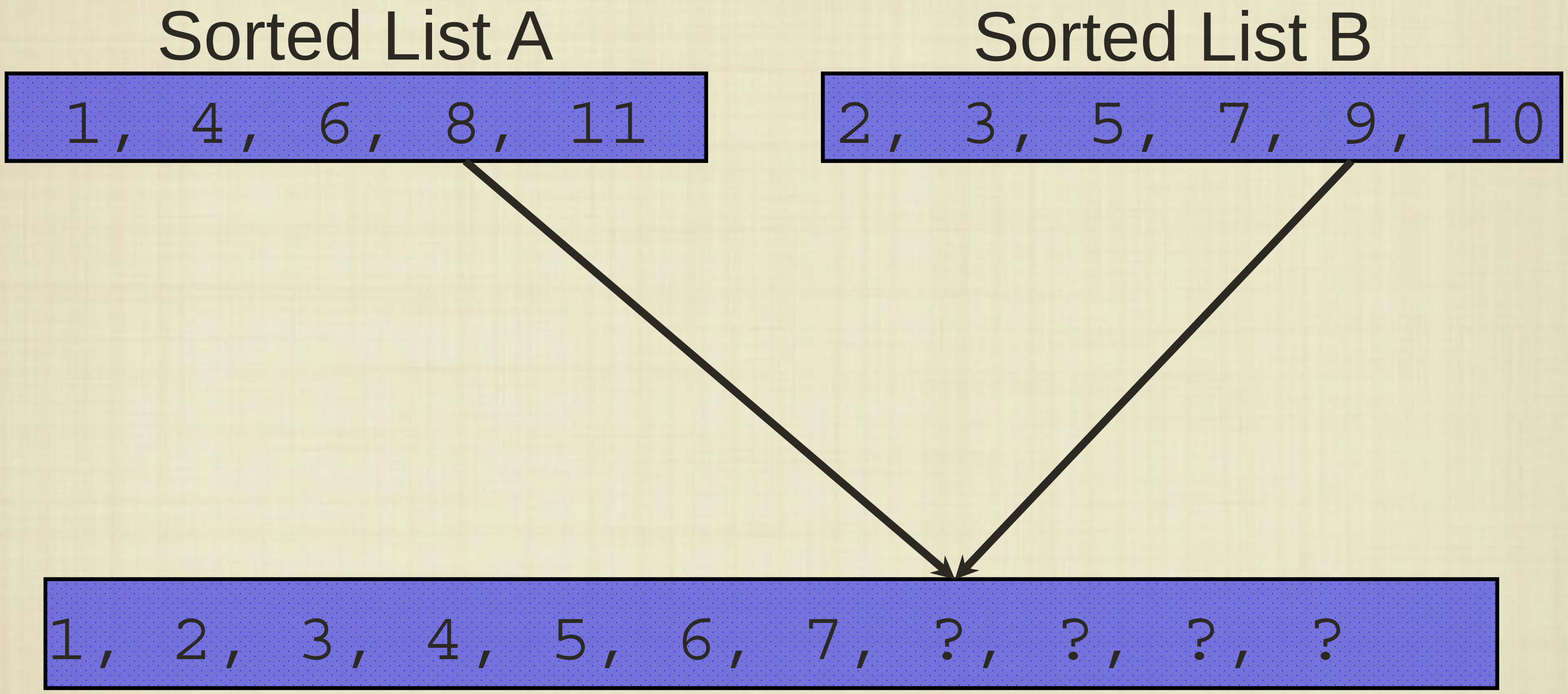

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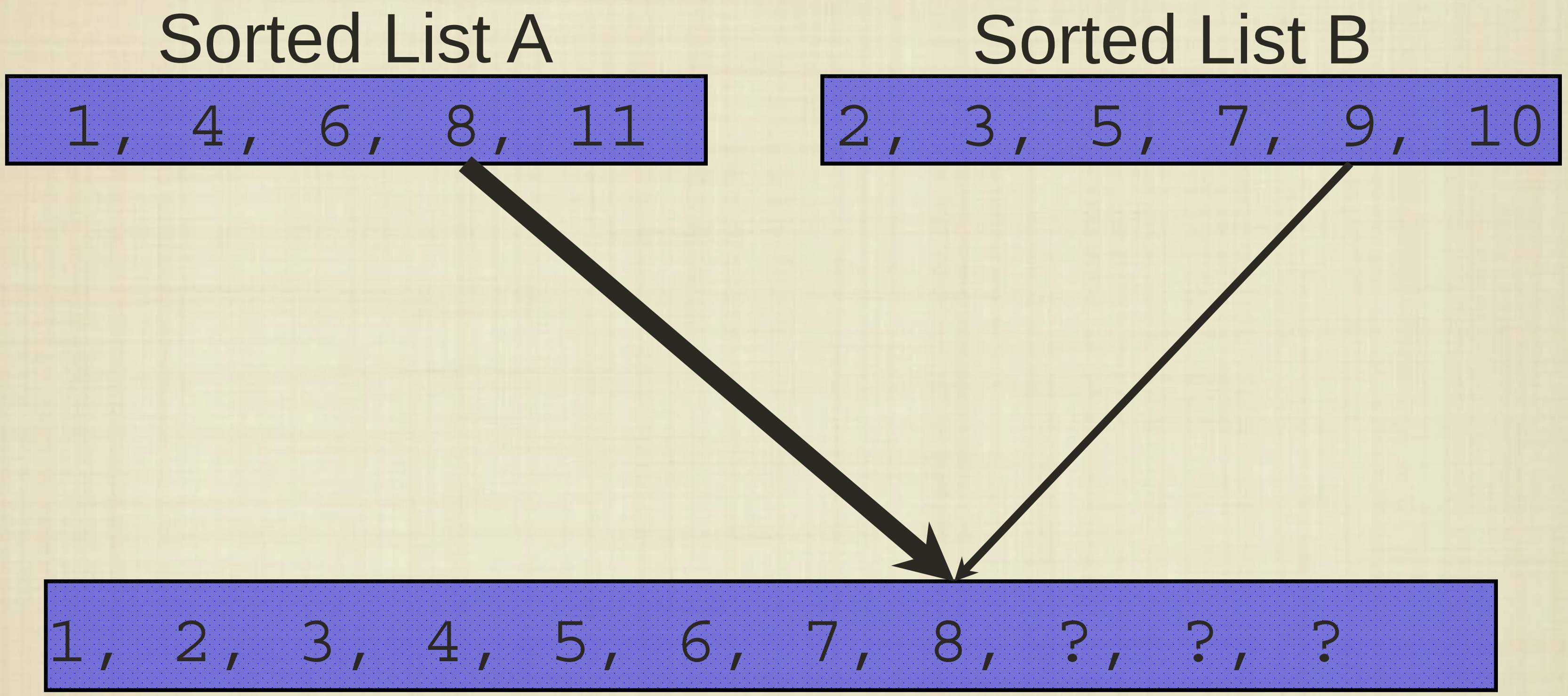

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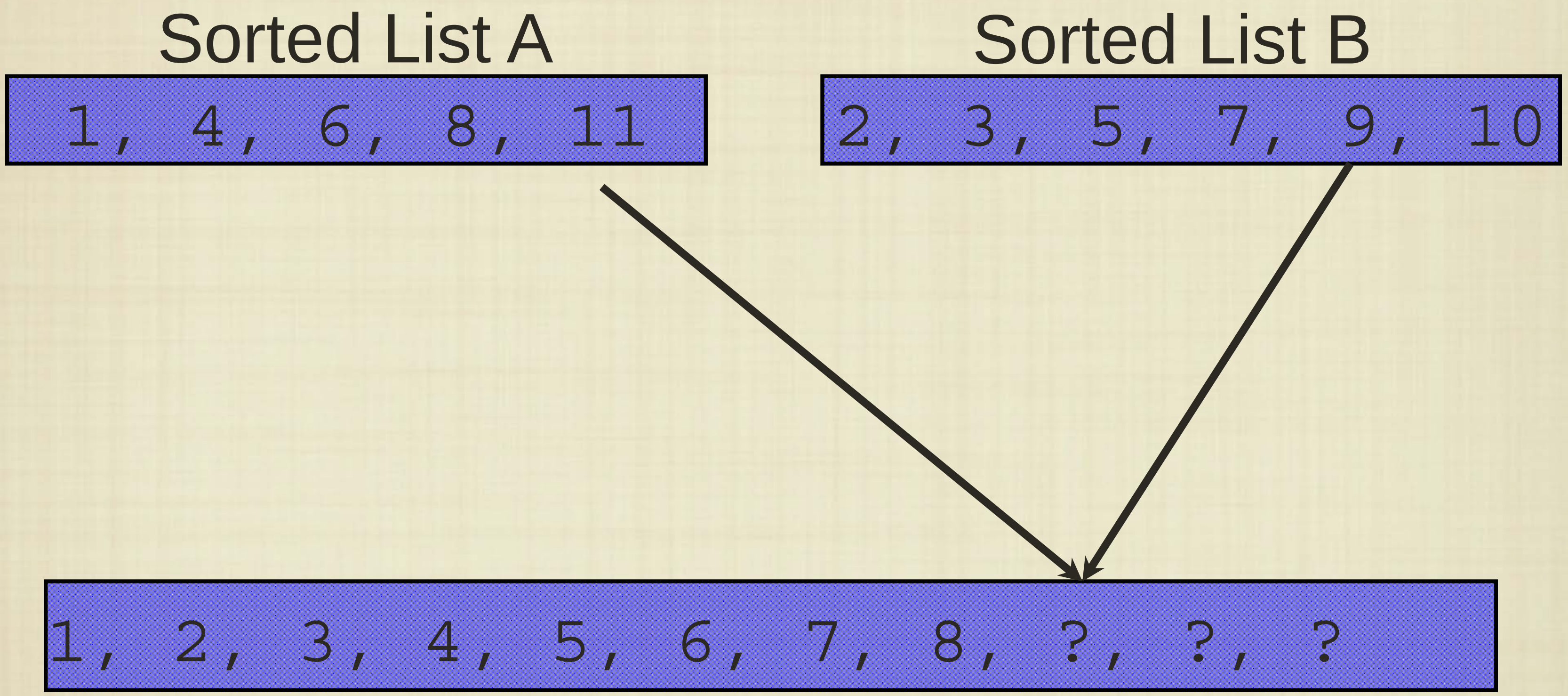

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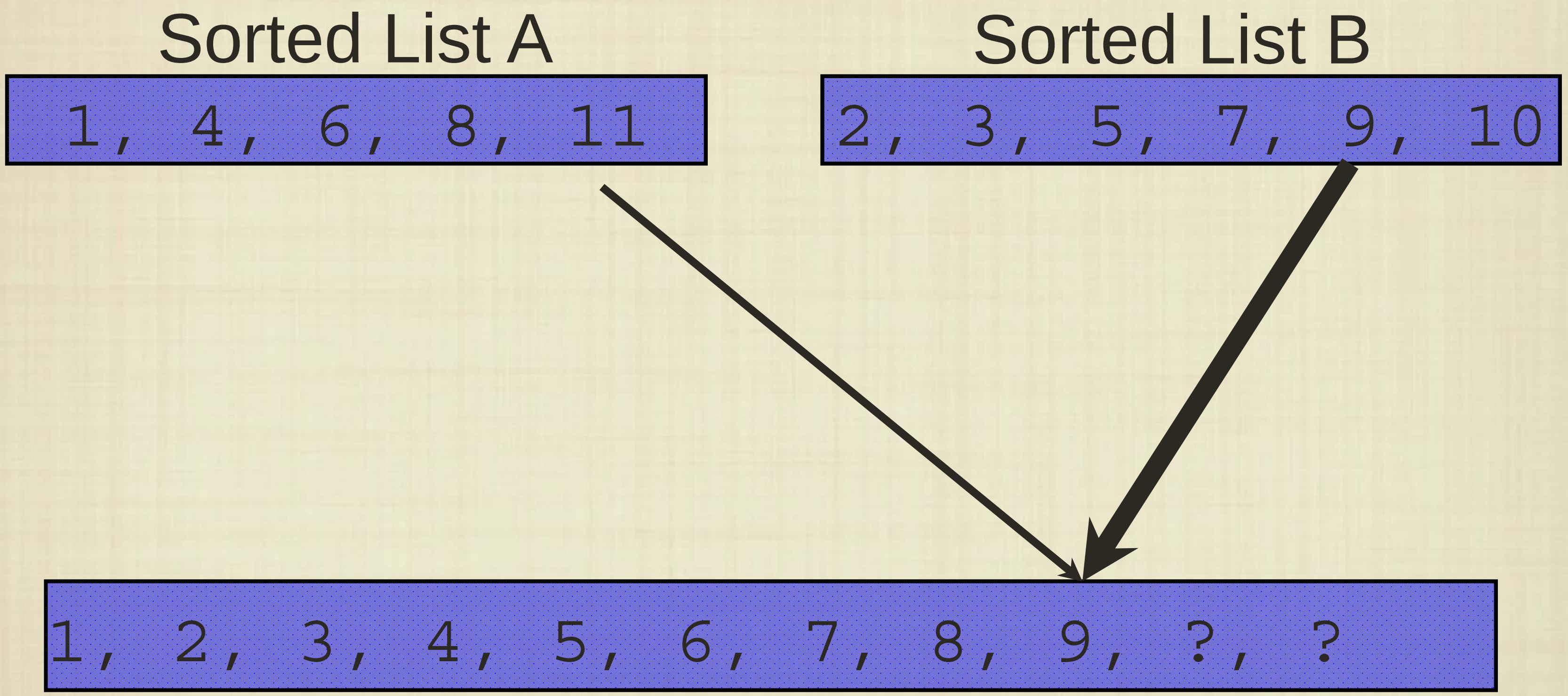

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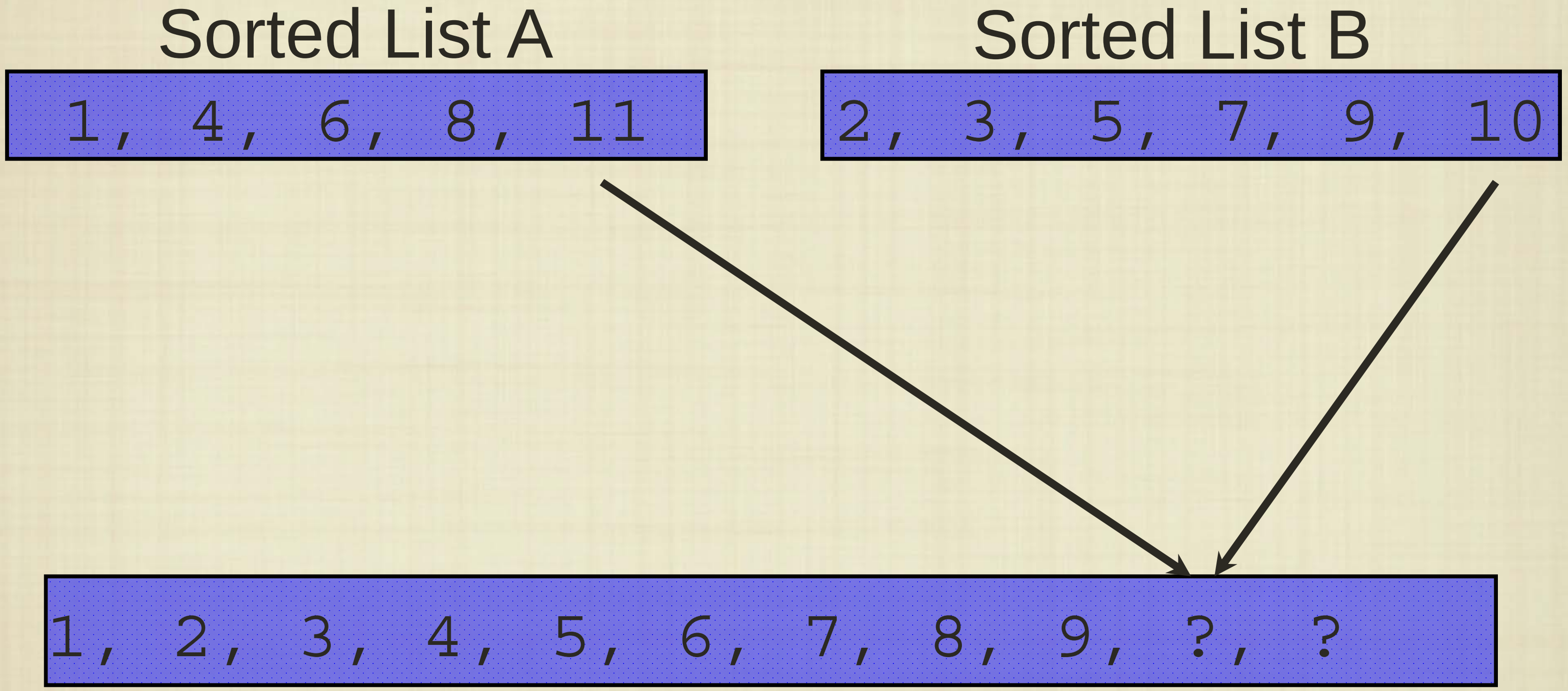

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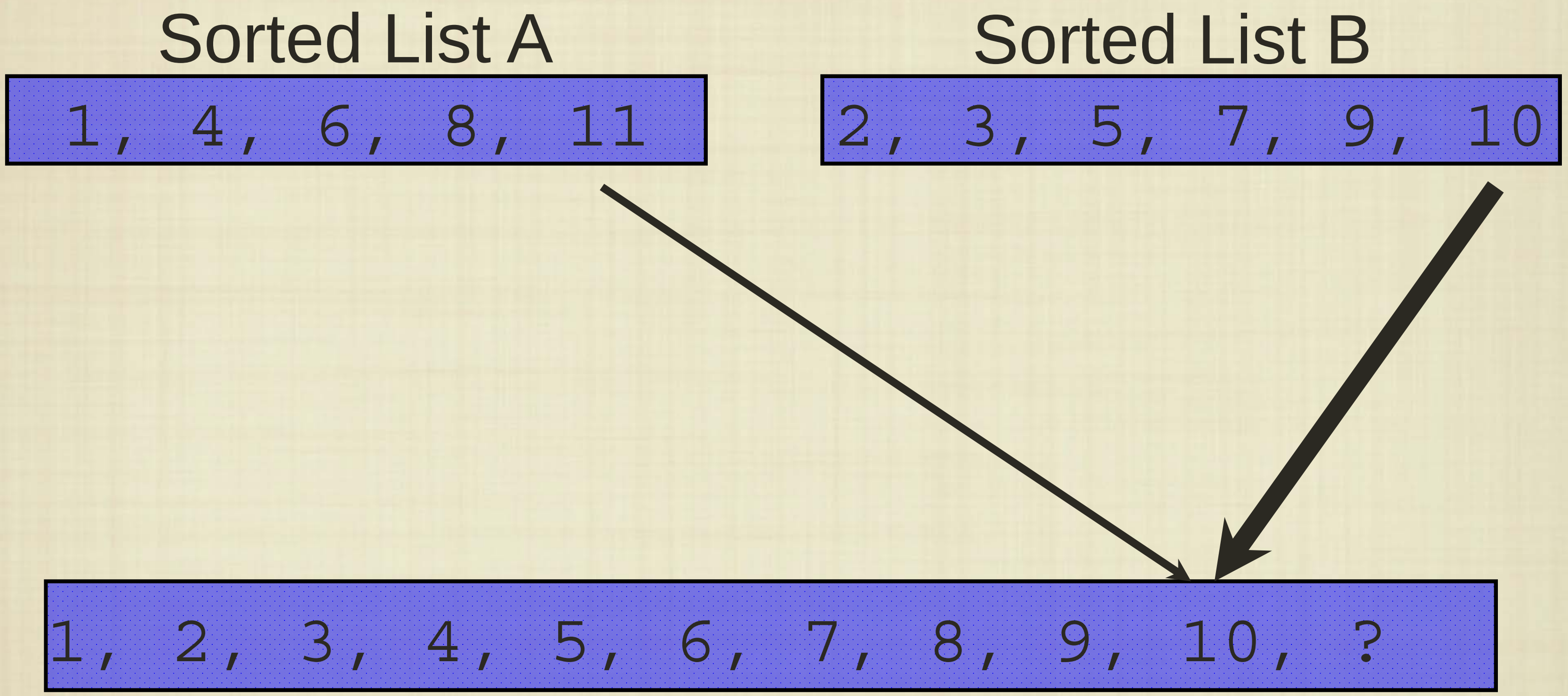

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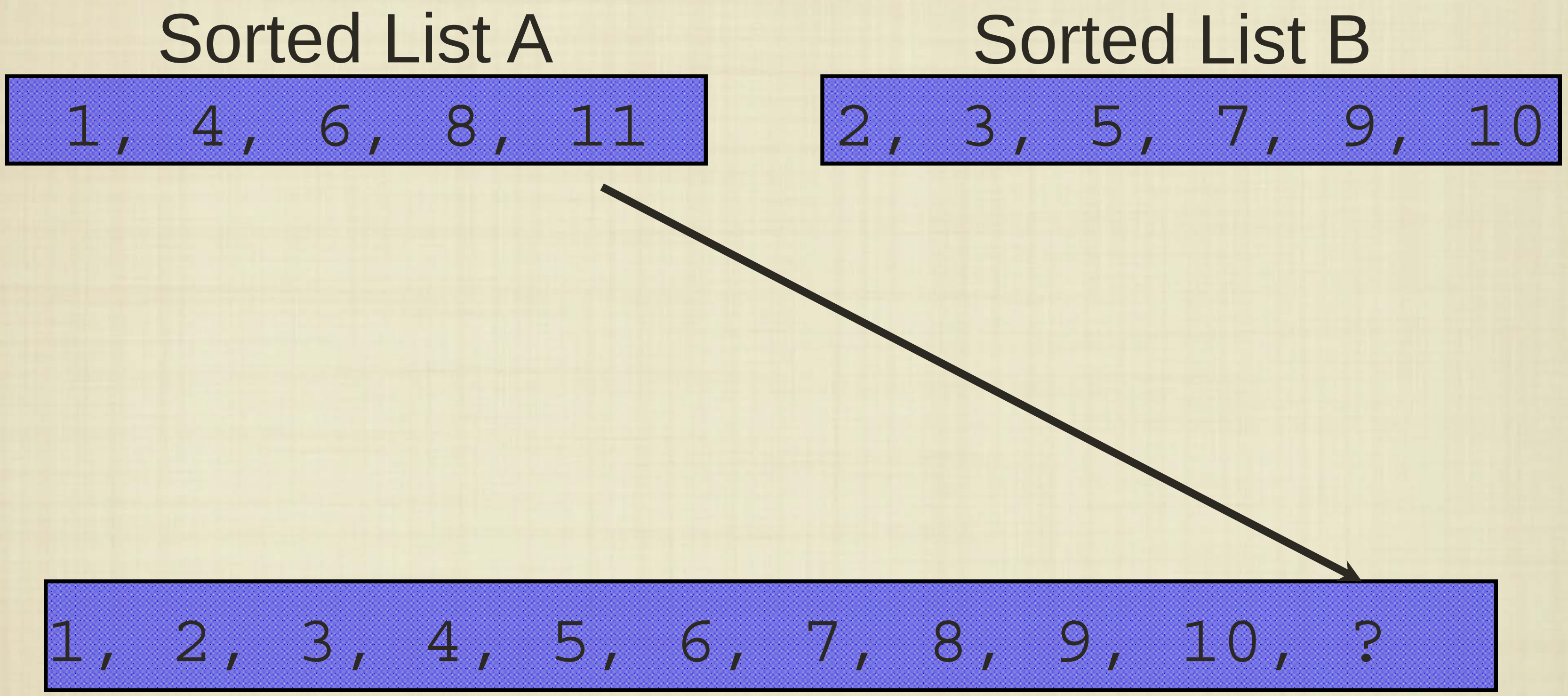

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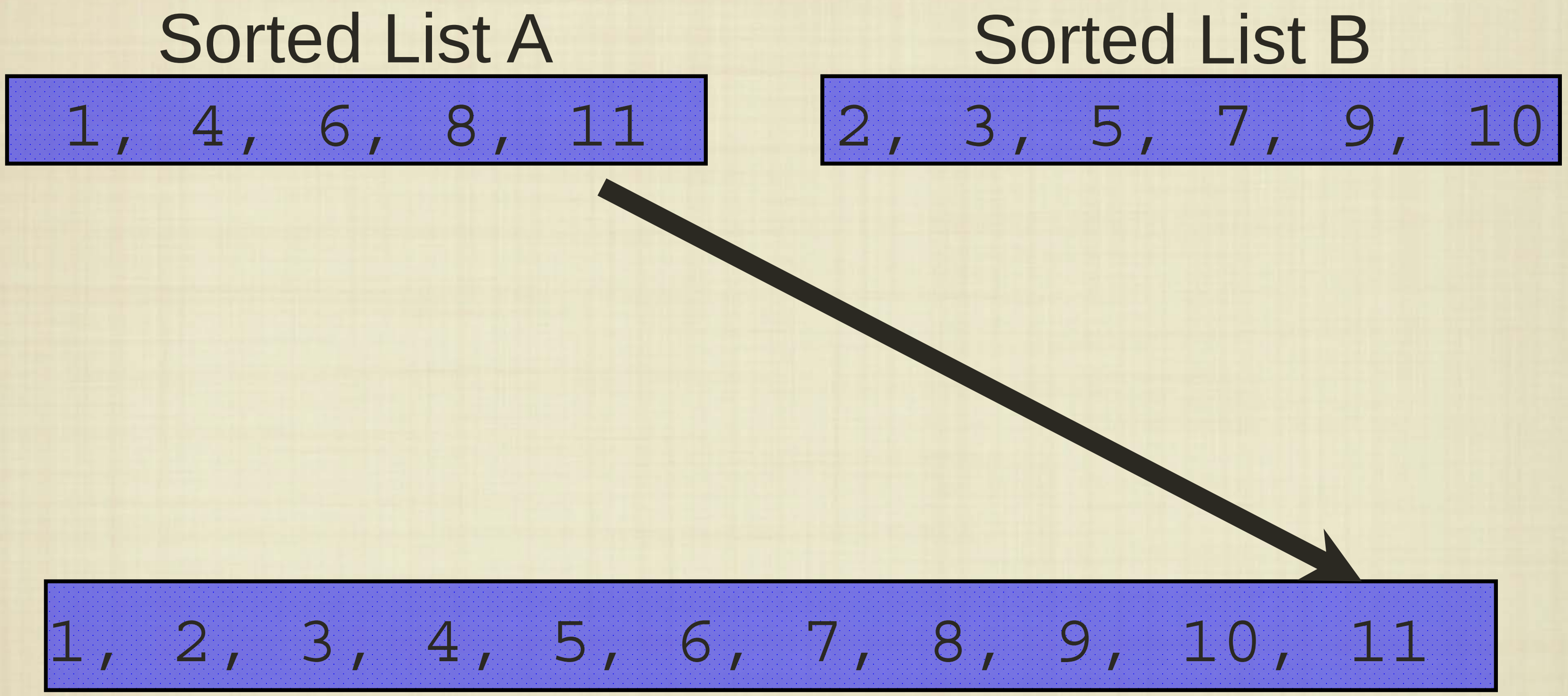

Merging Lists

Sorted List A

`1, 4, 6, 8, 11`

Sorted List B

`2, 3, 5, 7, 9, 10`

`1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11`

The key idea is to scan through both lists, while moving the smallest element to a new list. If we finish scanning either list, the rest of the other list is appended to the result.

Merge Sort

- Functional programming languages are ideally suited to implement recursive algorithms. How would we implement merge sort?

```
(define (merge-sort L)
(if (equal? (length L) 1)
    L
    (let ([mid (quotient (length L) 2)])
        (merge (merge-sort (take L mid))
             (merge-sort (drop L mid))))))
```

Assuming `merge` is correct, is `merge-sort` correct?

How do we implement `merge`?